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# NAVAL POSTGRADUATE SCHOOL

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## THESIS

NOISE CANCELLATION USING

ADAPTIVE ARRAYS

by

Constantinos G. Manikas

June 1984

Thesis Advisor:

H. Titus

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Noise Cancellation Using  
Adaptive Arrays

by

Constantinos G. Manikas  
Lieutenant, Hellenic Navy  
Hellenic Naval Academy, 1974

Submitted in partial fulfillment of the  
requirements for the degree of

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Author:

Approved by:

Thesis Advisor

Second Reader

Chairman, Department of Electrical and Computer Engineering

Dean of Science and Engineering

# ABSTRACT

All known adaptive beamformers utilize some form of automatic minimization of the mean square error. High adaptation rates though, exhibit a signal cancellation phenomenon leading to self-jamming by the adaptive antennas. This effect results from adaptive interaction between signal and interference (i.e., jammer) inputs simultaneously received by an adaptive antenna. This <sup>thesis</sup> ~~research~~ investigates various existing ways of adaptive beamforming for noise cancelling, and signal enhancement from simple Adaptive Noise Cancellers to Hard Constraint Adaptive Beamformers.

*Additional keywords: adaptive antennas, hard constrained beamformers, computer programs, adaptive filters, jamming.*

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## I. INTRODUCTION

Adaptive antennas have been under development in various forms during the past two decades, having thus far proven themselves capable of rejecting various jamming signals. Most high performance radar and communication systems being designed to work in jamming environments currently incorporate various forms of adaptive antennas. The concurrent development of spread spectrum techniques with the adaptive antennas provides a set of technologies for jam resistant systems. Both of the above technologies are compatible and frequently are used in the same system. The adaptive antenna is to attenuate the strong jamming signals as they appear at the front end of the receiver; then spread spectrum techniques are used to neutralize a large number of weak signals that may not be eliminated by the adaptive antenna, and to recover the desired signals.

The way an adaptive antenna works is to pass the signal contaminated with noise through a filter that tends to suppress the noise while leaving the signal unchanged. These kind of filters, known as adaptive filters, have the ability to adjust their own parameters automatically and their design requires little or no prior knowledge of the signal or noise characteristics.

Adaptive noise cancelling is a variation of optimal filtering that uses an auxiliary, or reference input. This input is then subtracted from the primary input, which is composed of both the desired signal and the undesired noise. This procedure of filtering and subtraction is controlled by an adaptive process, giving noise reduction which, depending on the particular application, can result in significant noise cancellation.

The design of an adaptive antenna has to retain signal preservation. Unfortunately some adaptive beamformers in use do not perform well in certain environments where correlated-signal conditions lead to partial or total cancellation of the desired signal within the antenna.

In that way, although our primary intention is the cancellation of all the undesired interferences, we create a self-jamming of the adaptive arrays by the so-called phenomenon of signal cancellation resulting from the adaptive interaction between the signal and the jammer which simultaneously are received by the adaptive antenna's elements.

## II. ADAPTIVE NOISE CANCELLER

### A. BASIC CONCEPT

The basic block configuration of an Adaptive Noise Canceller is shown in Figure 2.1. A signal,  $s$ , is transmitted over a channel to a sensor which also receives noise,  $n_0$ , uncorrelated with the signal. The combined signal-noise form the primary input to the canceller. A second sensor receives a noise,  $n_1$ , also uncorrelated with the signal, but correlated in some unknown way with the noise,  $n_0$ . This second sensor provides the so-called reference input of the canceller. Noise,  $n_1$ , is filtered to produce the output,  $y$ , that is as close as possible to a replica of  $n_0$ . Output,  $y$ , is now subtracted from the primary input to give the final system's output.

$$z = s + n_0 - y \quad (2.1)$$

The reference input,  $n_1$ , is processed by an adaptive filter which automatically adjusts its own impulse response. This adjustment is accomplished through an algorithm that responds to an error signal dependent on the filter's output. In this way, using the proper adaptive algorithm, the filter can operate under changing conditions which minimize the error signal.

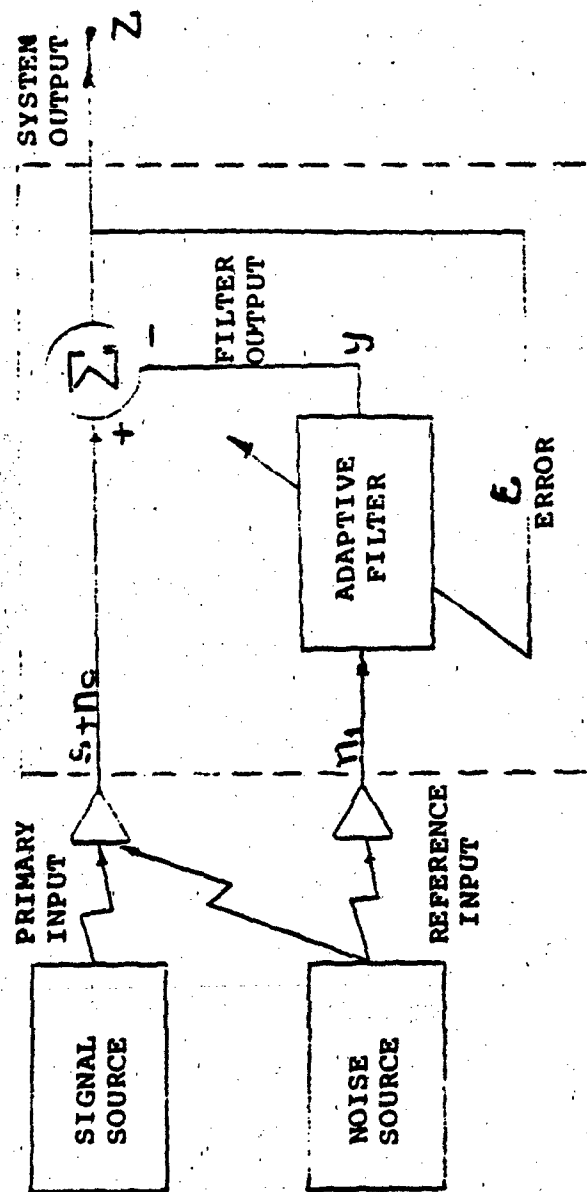


Figure 2.1. Basic Configuration of the A.N.C.

The error signal,  $e$ , is feeding the system's output back to the adaptive filter in order to adjust the filter through a LMS (Least Mean Squares) adaptive algorithm and minimizes the total system output power. In other words, in an adaptive noise canceller the output forms an error signal for the adaptive process.

### B. ANALYTICAL MODEL--NOTCH FILTER RESPONSE

In order to analyze the adaptive filter, we choose a pure sinusoidal for reference input.

$$x_k = C \cos(\omega_0 t + \theta) \quad (2.2)$$

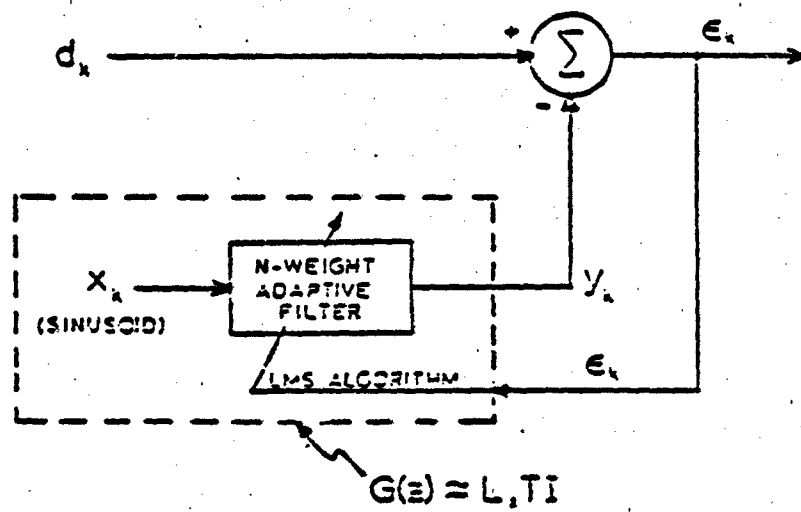
Glover [Ref. 1] has shown that under this condition the dashed box of Figure 2.2(a) can be approximated by Figure 2.2(b).

The conventional way of eliminating such sinusoidal interferences is through the use of a notch filter. Figure 2.3 shows a single frequency noise canceller, with two adaptive weights, which are updated through the equations:

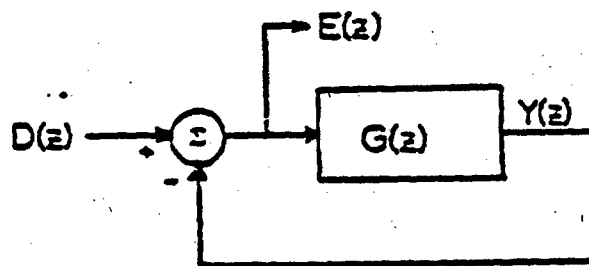
$$w_{1J+1} = w_{1J} + 2\mu e_J x_{1J} \quad (2.3)$$

$$w_{2J+1} = w_{2J} + 2\mu e_J x_{2J} \quad (2.4)$$

The sampled reference inputs are:



(a)



(b)

Figure 2.2. Analytical Model of the A.N.C.

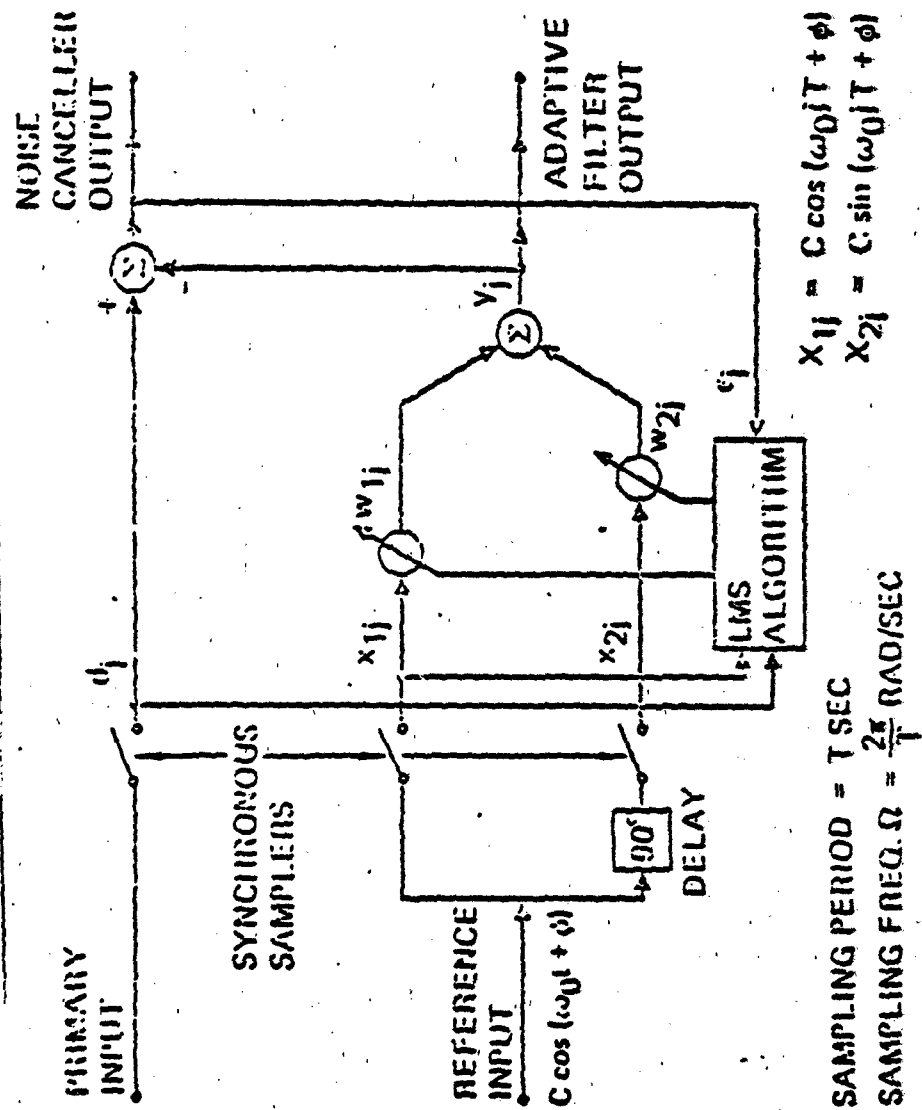


Figure 2.3. Two-Weight Noise Canceller



$$x_{1J} = C \cos(\omega_0 J T + \phi) \quad (2.5)$$

$$x_{2J} = C \sin(\omega_0 J T + \phi) \quad (2.6)$$

Widrow [Ref. 2] and [Ref. 3] has shown that the single frequency noise canceller has the properties of a notch filter<sup>1</sup> at the reference frequency,  $\omega_0$ , having the transfer function:

$$Y(z)/X(z) = H(z)$$

where

$$H(z) = \frac{z^2 - 2z \cos(2\pi\omega_0 z^{-1}) + 1}{z^2 - 2(1 - \mu C^2)z \cos(2\pi\omega_0 z^{-1}) + 1 - 2\mu C^2} \quad (2.7)$$

The zeros of the above transfer function are located in the z-plane at

$$z = e^{\pm j2\pi\omega_0 z^{-1}} \quad (2.8)$$

and are inside the unit circle.

The poles, also inside the unit circle, are located at

$$z = (1 - \mu C^2) \cos(2\pi\omega_0 z^{-1}) \pm j[(1 - 2\mu C^2) - (1 - \mu C^2) \cos^2(2\pi\omega_0 z^{-1})]^{1/2} \quad (2.9)$$

---

<sup>1</sup>Detailed explanation is given in Appendix A.

Figure 2.4 indicates the location of poles and zeros in the  $z$ -plane. Since the zeros lie inside the unit circle, the depth of the notch in the transfer function is infinite at the interference frequency,  $\omega_0$ . The sharpness of the notch is determined by the closeness of the poles to the zeros.

Figures 2.5, 2.6 and 2.7 are magnitude plots of the transfer function  $H(z)$  using as  $C$  the value of one, different values of the adaptation constant  $\mu$ .

#### C. NOISE CANCELLATION IN THE NARROWBAND CASE

Consider the case in which both signal and jammer (i.e., interference) are pure sinusoids of known frequencies,  $f_s$ ,  $f_j$ , where

$$f_s \neq f_j \quad (2.10)$$

Figure 2.8 shows the frequency spectra of both noise and jammer, and Figure 2.9 indicates the same spectra after being filtered by a notch filter with cutoff frequency the same as the one of the jammer,  $f_j$ .

Comparing the two preceding Figures 2.8 and 2.9, we see how effective the notch filter is in eliminating sinusoidal interferences. Of course, pure sinusoidal interferences are seldom found in nature, and still the main disadvantage is that noise and jammer have to be of different frequencies.

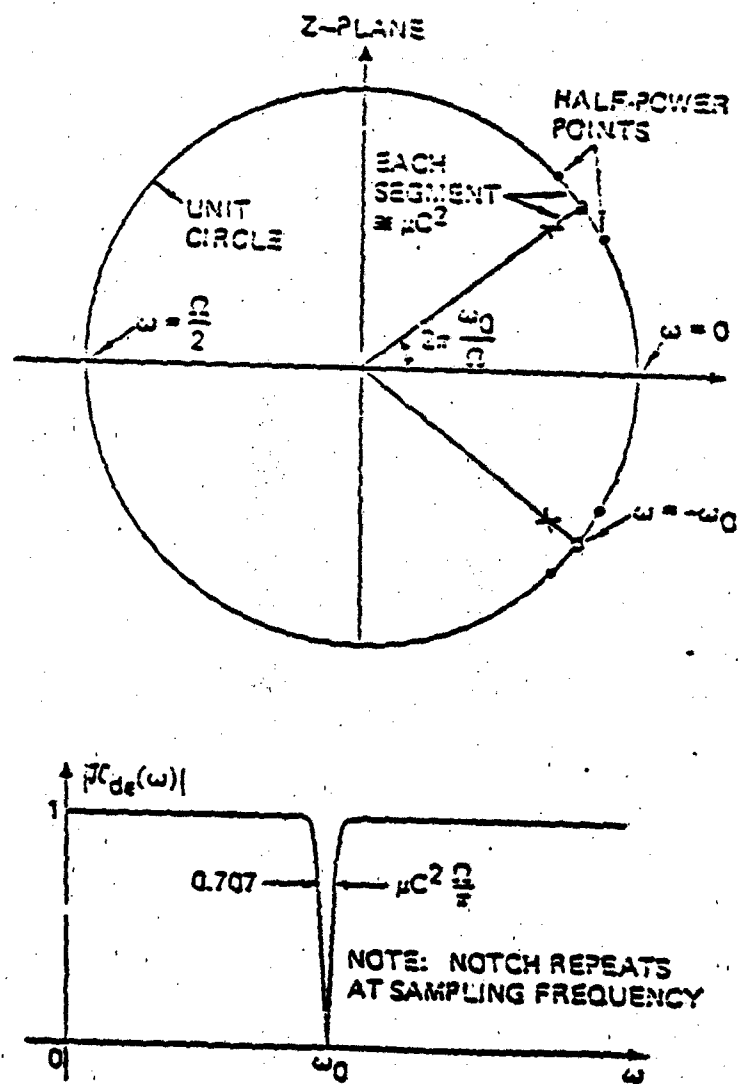


Figure 2.4. Pole-Zero Configuration

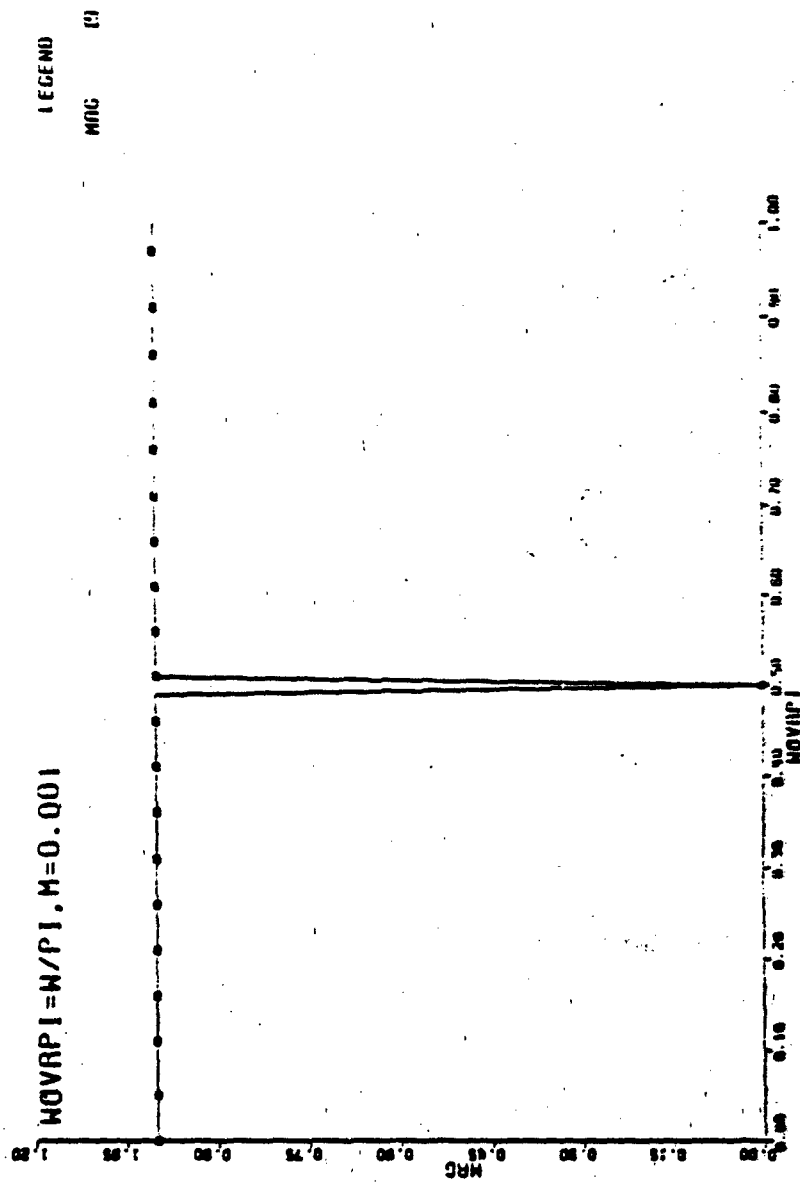


Figure 2.5. Frequency Response of a Notch Filter ( $\mu = 0.001$ )

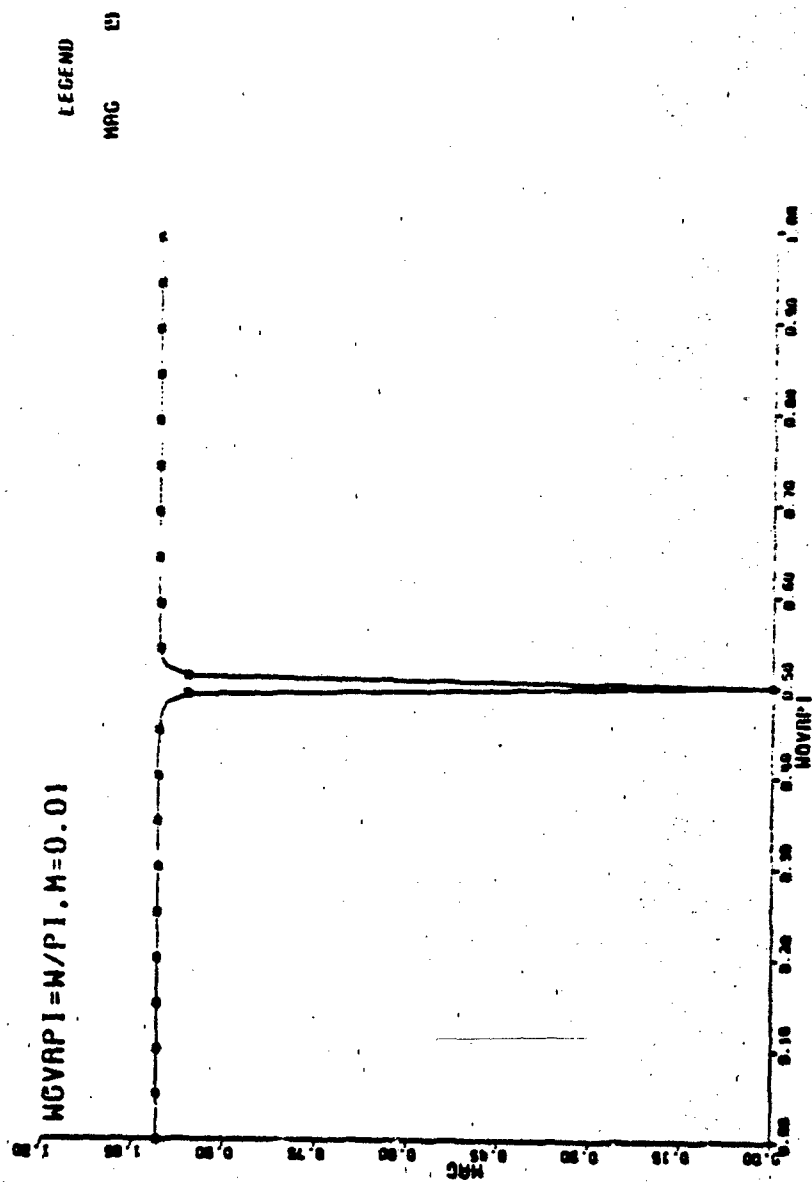


Figure 2.6. Frequency Response of a Notch Filter ( $\mu = 0.01$ )

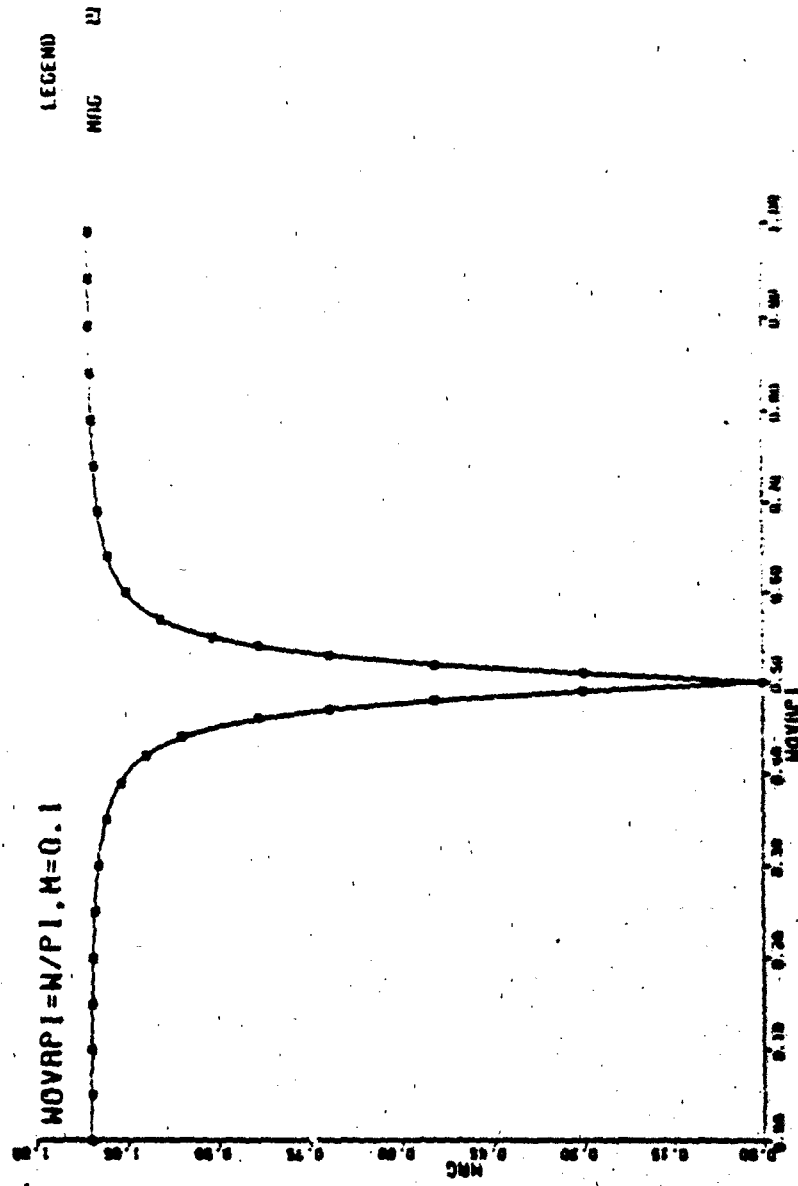


Figure 2.7. Frequency Response of a Notch Filter ( $\mu = 0.1$ )

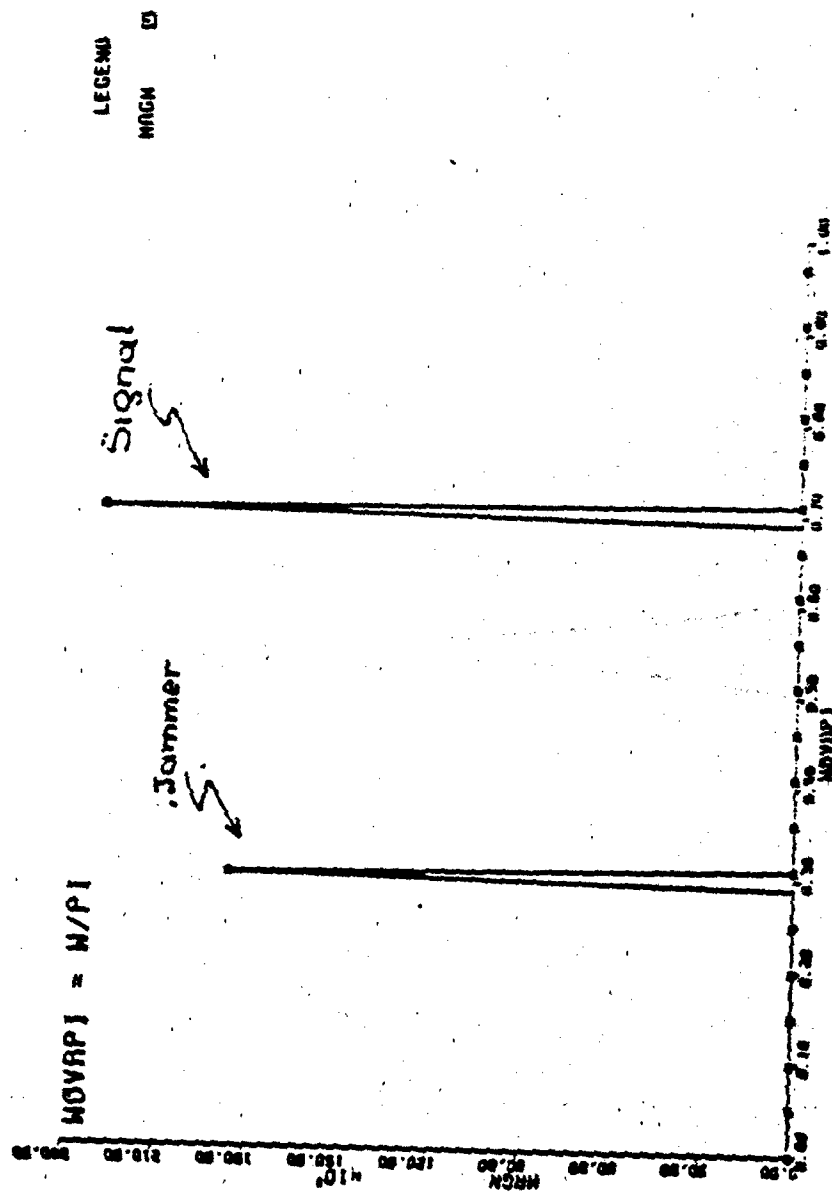


Figure 2.8. Frequency Response of Sinusoidal Signal-Jammer

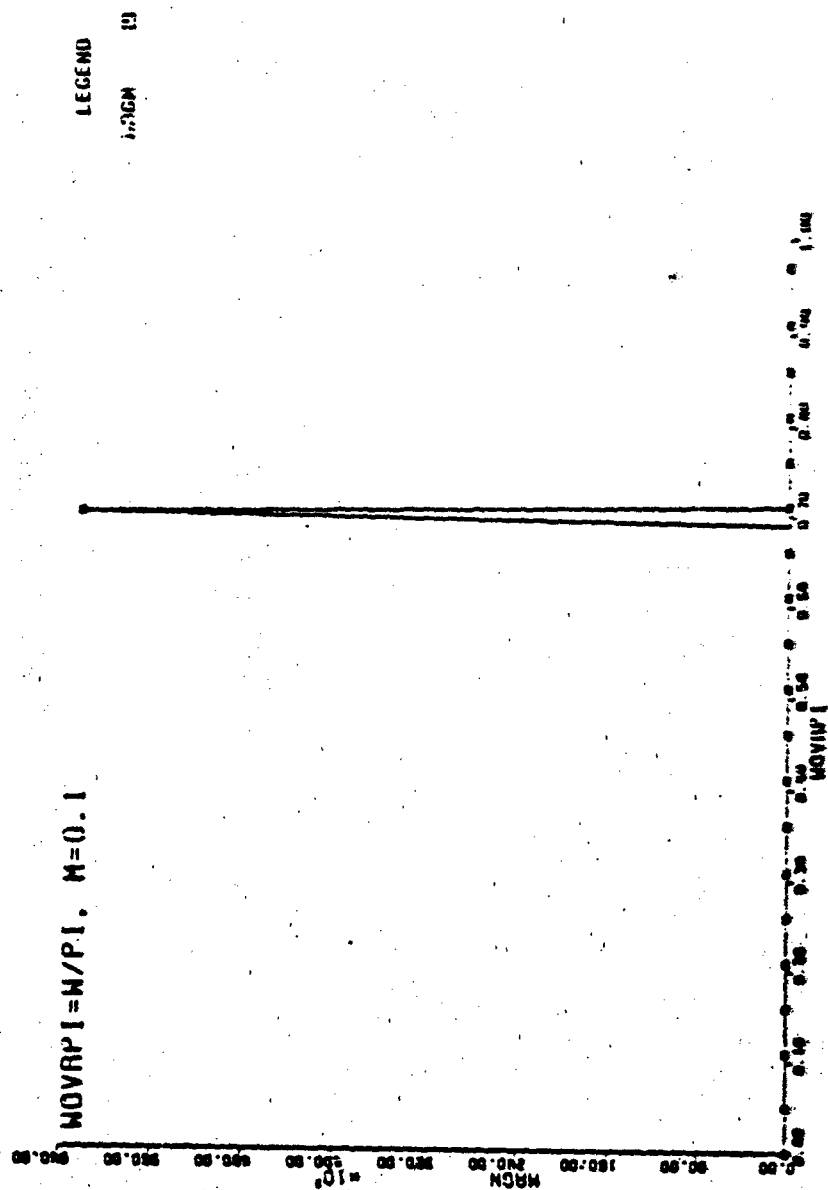


Figure 2.9. Frequency Response of Two Filtered Sinusoids



### III. FROST BASED ADAPTIVE BEAMFORMER

#### A. CONSTRAINED LEAST MEAN SQUARES ALGORITHM

Frost [Ref. 4] proposed an algorithm, the L.M.S. algorithm, for adjusting an array of sensors in real time to respond to a desired signal while discriminating against noises. Figure 3.1 illustrates the basic set-up for implementing an L.M.S. algorithm. It is assumed that the desired signal arrives in plane waves from a chosen direction called the look direction.

The algorithm iteratively adapts weights in order to minimize noise power at the array output while maintaining a chosen frequency response in the look direction. Requirements for the algorithm is a priori knowledge of the direction of arrival and the frequency band of interest. During the adaptive process, the algorithm progressively learns the statistics of noise arriving from all directions except the look direction.

A major advantage of the constrained L.M.S. algorithm is that it has a self-correcting feature, permitting it to operate for arbitrarily long periods of time in a digital computer implementation without deviating from its constraints because of cumulative roundoff or truncation errors.

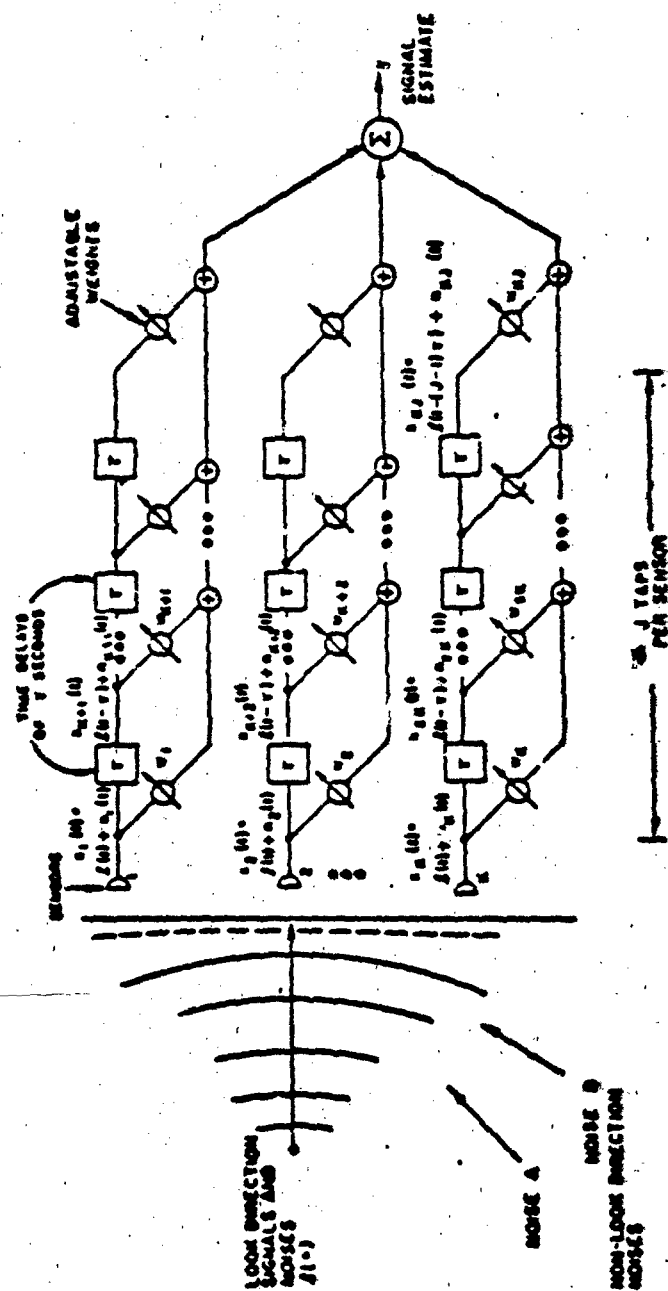


Figure 3.1. Basic Configuration of an Adaptive Beamformer

Noise arriving from the look direction may be filtered out by a suitable choice of the frequency response characteristics in that direction or by some external means.

#### B. BASIC PRINCIPLE OF THE CONSTRAINTS

L.M.S. Algorithm maintains a chosen frequency response in the look direction while minimizing output noise power because of a simple relation between the look direction frequency response and the weights in the array of Figure 3.1. Assuming that the look direction is chosen perpendicular to the line of sensors, identical signal components, arriving on a plane wavefront parallel to the line of sensors, appear at the first taps simultaneously and parade in parallel down the tapped delay lines<sup>2</sup> following each sensor. However, noise waveforms arriving from other than the look direction will not, generally, produce equal voltage components on any given vertical column of taps. The voltages, signal plus noise, at each tap are multiplied by the tap weights and added to form the array output. Thus, as far as the signal is concerned, the array processor is equivalent to a single tapped delay line in which each weight is equal to the sum of the weights in the corresponding vertical column of the processor, as indicated in Figure 3.2. These summation weights in the equivalent

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<sup>2</sup>Appendix B describes the tapped delay line filter.

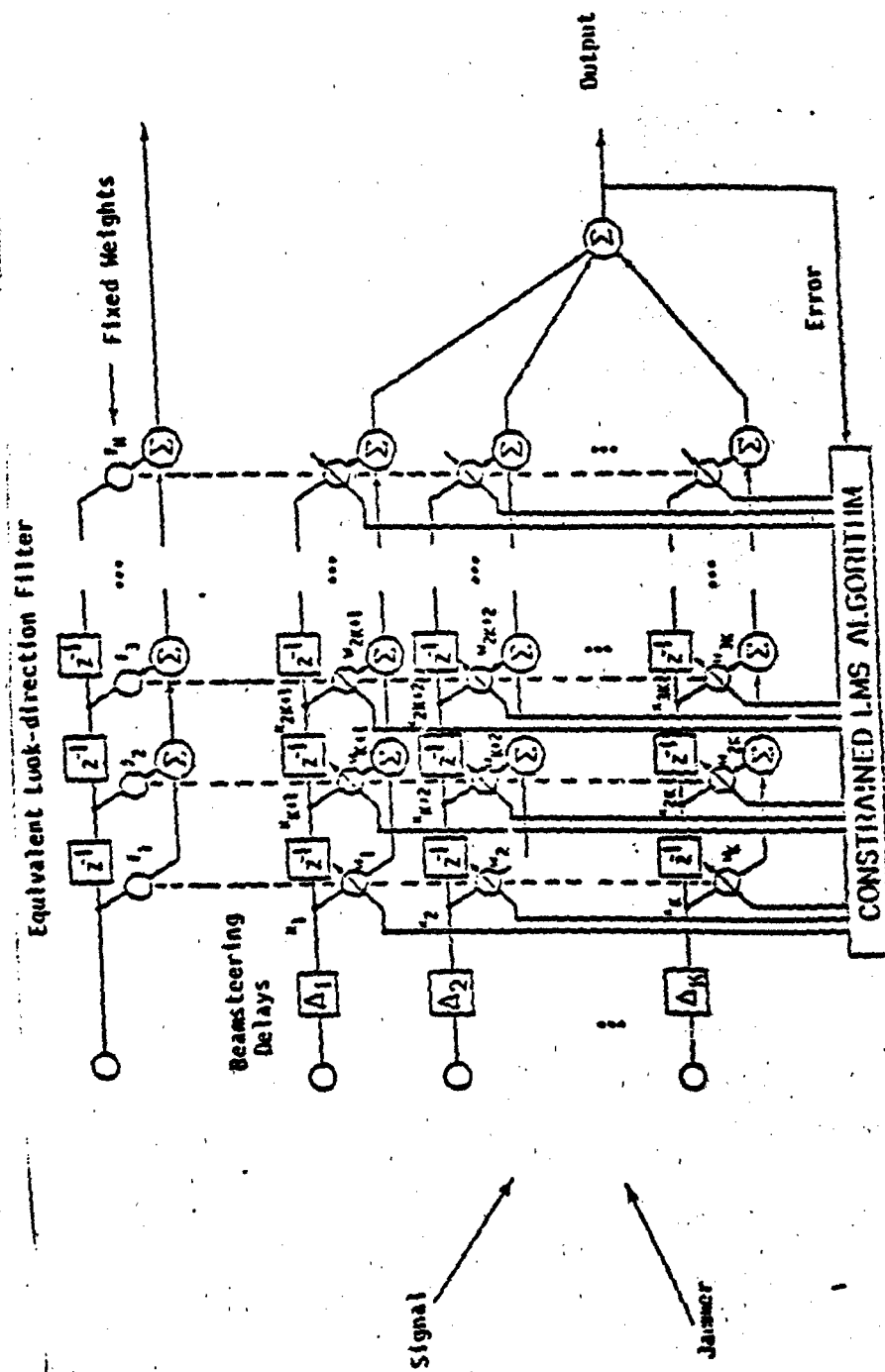


Figure 3.2. PROST Based Adaptive Beamformer

tapped delay line must be selected so as to give the desired frequency response characteristics in the look direction.

If the look direction is chosen to be other than the one perpendicular to the line of sensors, then the array can be steered either mechanically, or electrically, by the addition of steering time delays placed immediately after each sensor.

Now consider the array problem of Figure 3.2.

In vector notation the sample tap voltages are identified as

$$x^T(j) \triangleq [x_1(j), x_2(j), \dots, x_{k_j}(j)] \quad (3.1)$$

and the sampled weights as

$$w^T(j) \triangleq [w_1(j), w_2(j), \dots, w_{k_j}(j)] \quad (3.2)$$

where  $j$  represents the sample number. The filter outputs are summed to form the beamformer output

$$y(j) \triangleq w^T(j) \cdot x(j) = x^T(j) \cdot w(j) \quad (3.3)$$

If we denote by  $C$  the constraint matrix, defined as

$$C = [c_1, c_2, \dots, c_j] \quad (3.4)$$

a  $KJ \times J$  dimensional matrix, that conveys the beamformer structural information needed to implement the constraints, then the constrained L.M.S. problem may be stated as

$$\text{Minimize } E[y^2(j)] = E[W^T \cdot X(j) X^T(j) \cdot W] \quad (3.5)$$

$$\text{subject to } C^T W = \hat{L} \quad (3.6)$$

where  $\hat{L}$  is the look-direction response vector defined as

$$\hat{L} \triangleq [f_1, f_2, \dots, f_J] \quad (3.7)$$

The weights are updated using the constrained L.M.S. algorithm, which for the case of Figure 3.2 (assuming two sensors), can be written as

$$\begin{aligned} w_1(j+1) &= (1/2)[w_1(j) - \mu y(j)x_1(j)] \\ &\quad - (1/2)[w_2(j) - \mu y(j)x_2(j)] + (f_1/2) \end{aligned} \quad (3.8)$$

$$\begin{aligned} w_2(j+1) &= (1/2)[w_2(j) - \mu y(j)x_2(j)] \\ &\quad - (1/2)[w_1(j) - \mu y(j)x_1(j)] + (f_1/2) \end{aligned} \quad (3.9)$$

$$w_3(j+1) = (1/2)[w_3(j) - u_Y(j)x_3(j)] \\ - (1/2)[w_4(j) - u_Y(j)x_4(j)] + (f_2/2) \quad (3.10)$$

$$w_4(j+1) = (1/2)[w_4(j) - u_Y(j)x_4(j)] \\ - (1/2)[w_3(j) - u_Y(j)x_3(j)] + (f/2) \quad (3.11)$$

The performance that a FROST based adaptive beamformer exhibits is unity gain and flat frequency response in the look direction. A deep null must be placed at the jammer's frequency. Figure 3.3 is a plot of the signal direction frequency response, and Figure 3.4 is a plot of the jammer-direction frequency response. Figure 3.5 indicates the beampattern of the FROST beamformer, confirming that a deep null has been placed on the jammer.

#### C. NOISE CANCELLATION IN THE WIDEBAND CASE

The first case for examination is the one of a sinusoidal jammer of frequency,  $f_J$ , and a broadband signal of center frequency,  $f_S$ , where

$$f_J = f_S \quad (3.12)$$

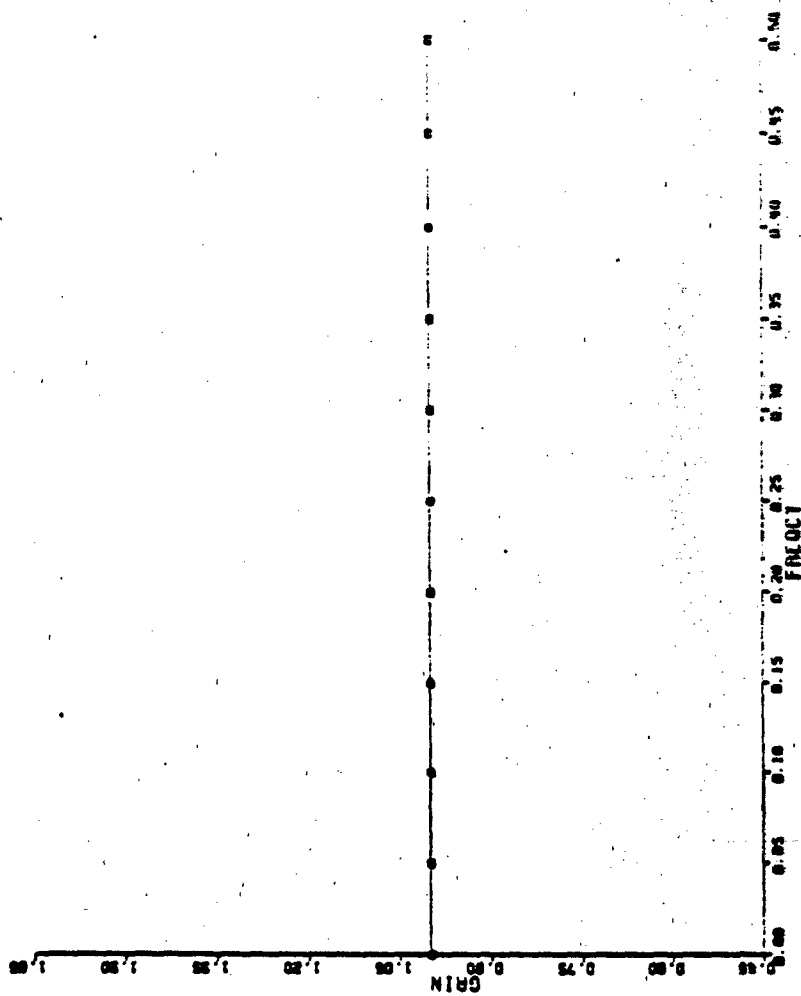


Figure 3.3. Signal-Direction Frequency Response



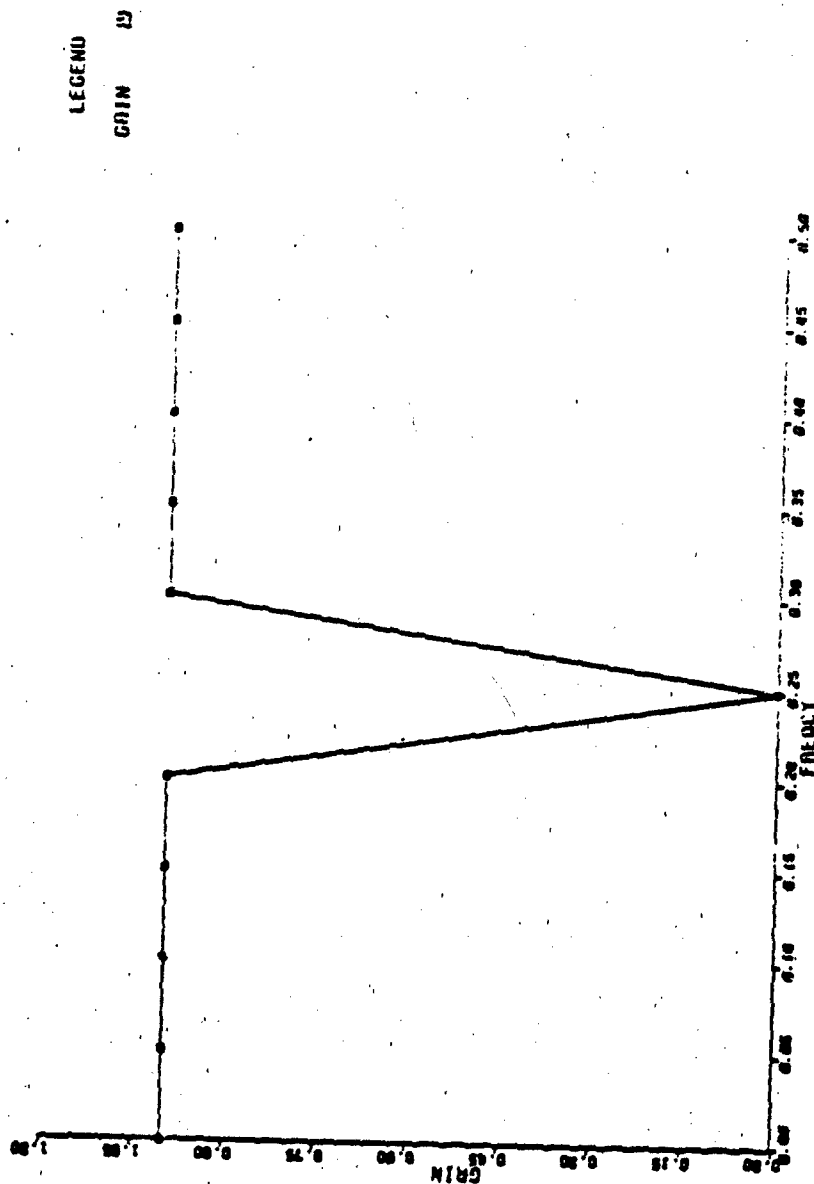


Figure 3.4. Jammer-Direction Frequency Response

## BROADSIDE ARRAY PATTERN

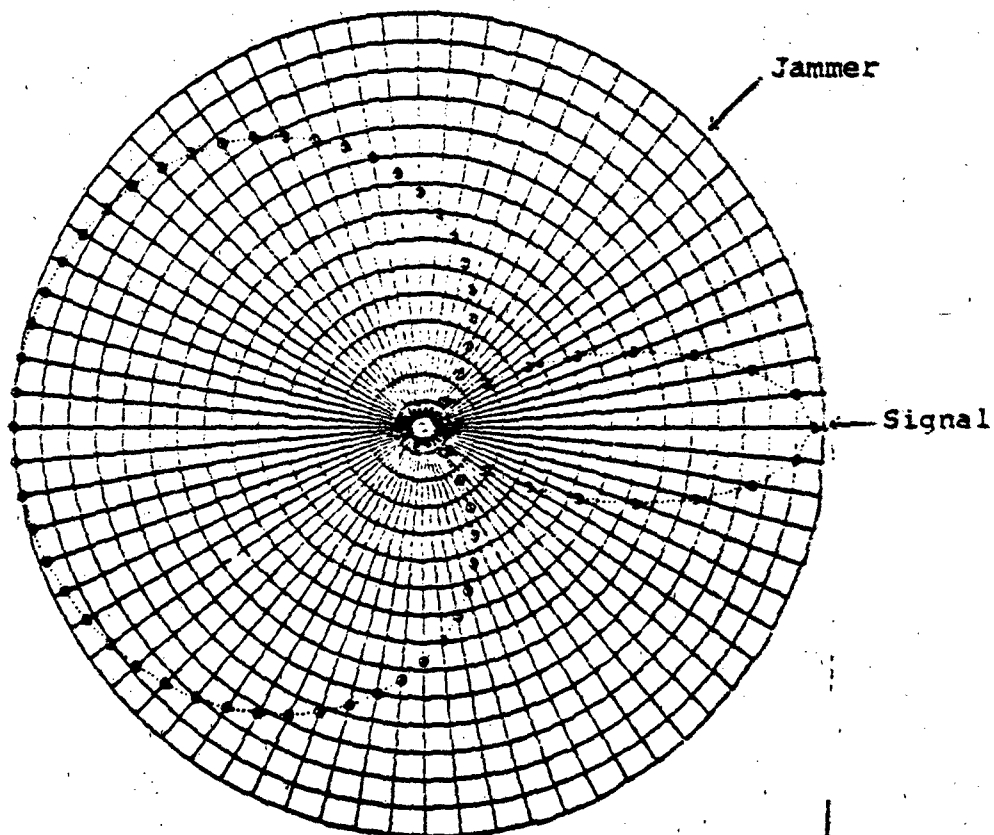


Figure 3.5. FROST A.B.F. Beampattern

Figure 3.6 represents the spectrum of the signal, since Figure 3.7 indicates the spectrum of the stronger sinusoidal jammer. Figures 3.8, 3.9 and 3.10 represent results of passing the above described signal and noise through notch filters, each one employing a different adaptation constant.

Comparing the preceding three figures we see how important the adaptation constant,  $\mu$ , is and how it can lead to signal aliasing by choosing it to have bigger than proper magnitude.

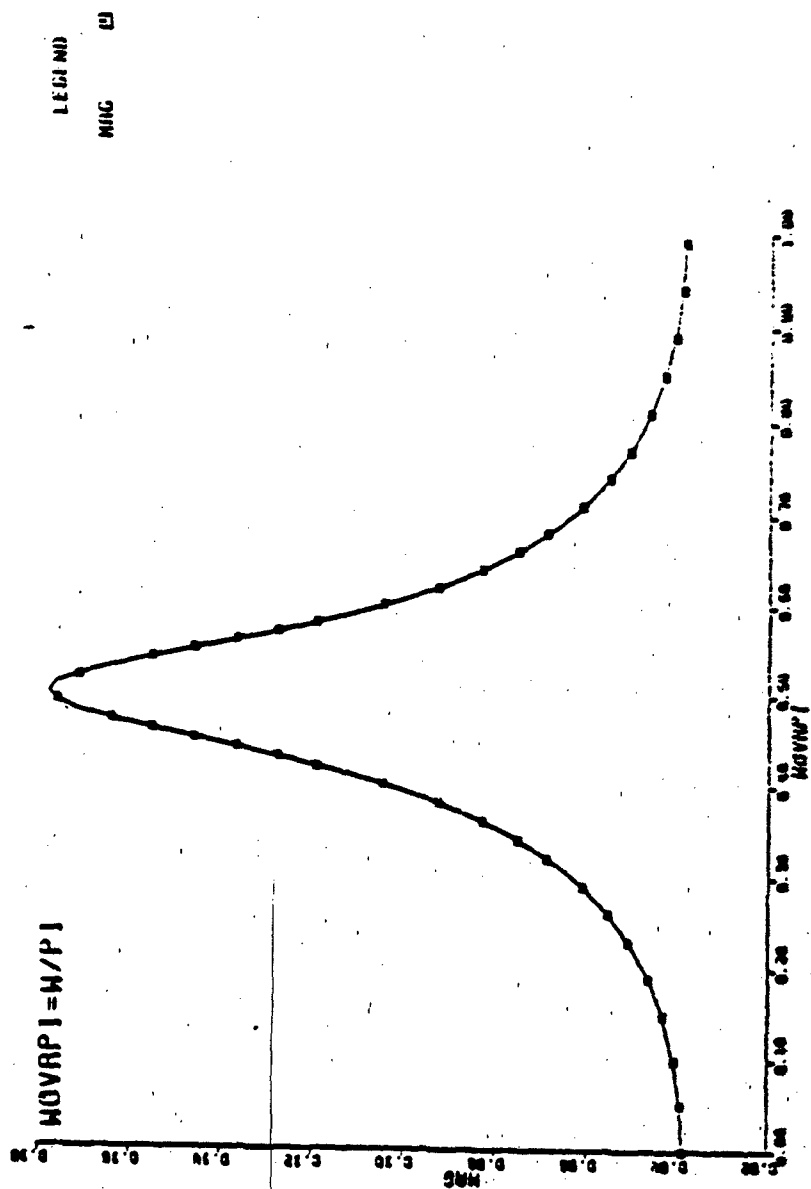
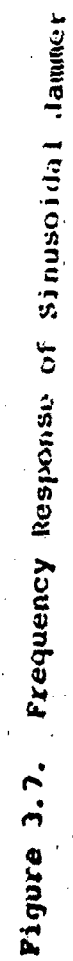


Figure 3.6. Frequency Response of a Broadband Signal



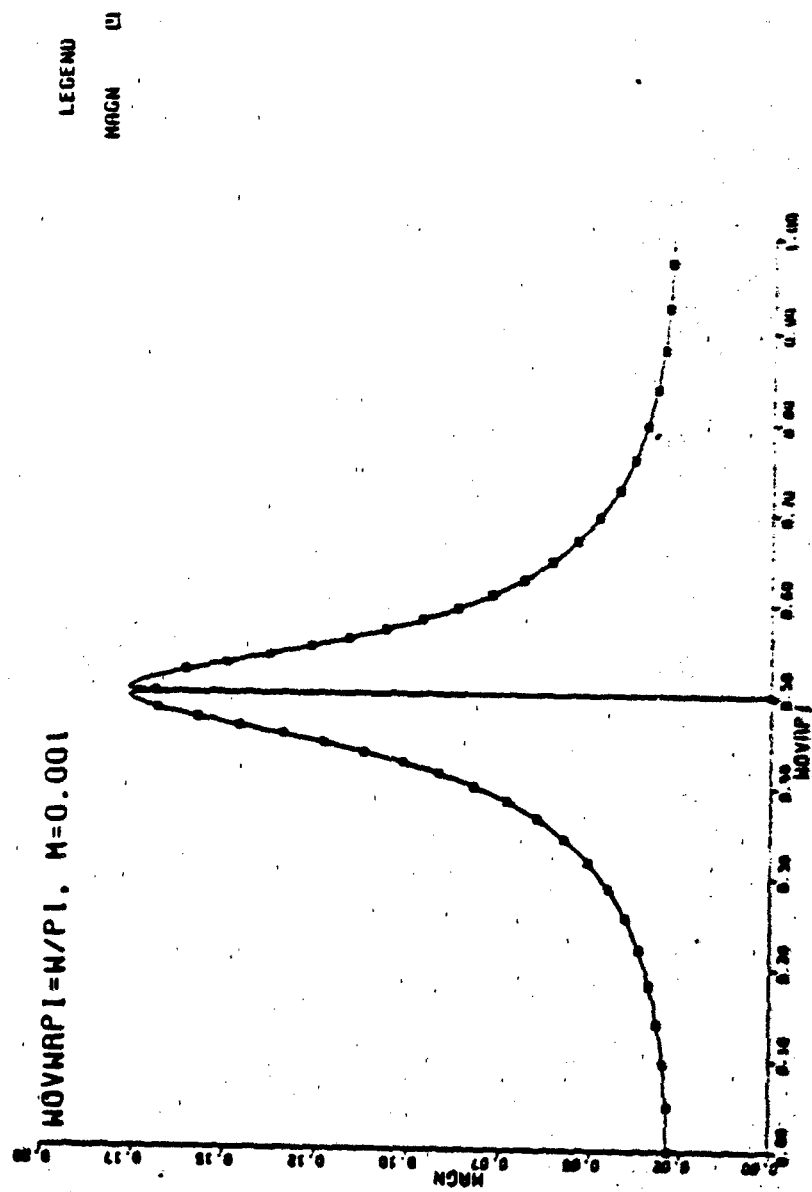


Figure 3.8. Filtered Signal-Jammer ( $\mu = 0.001$ )

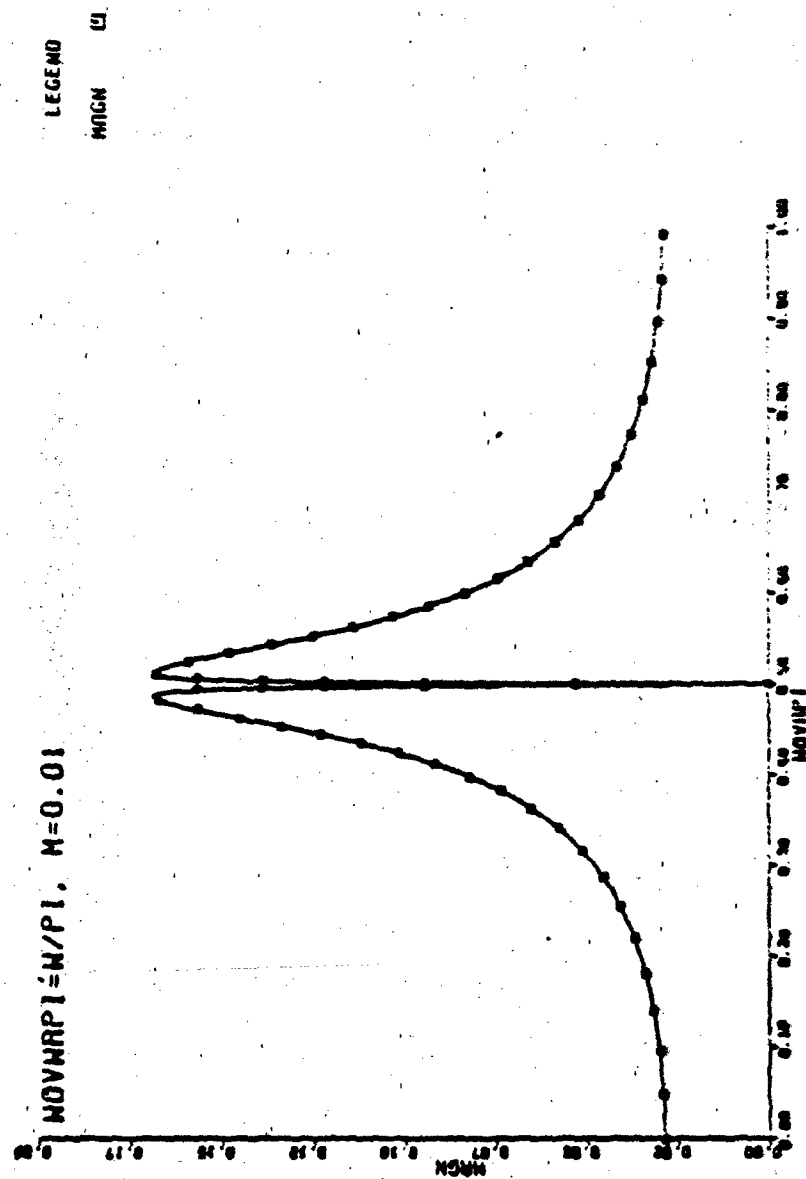


Figure 3.9. Filtered Signal-Jammer ( $\mu = 0.01$ )

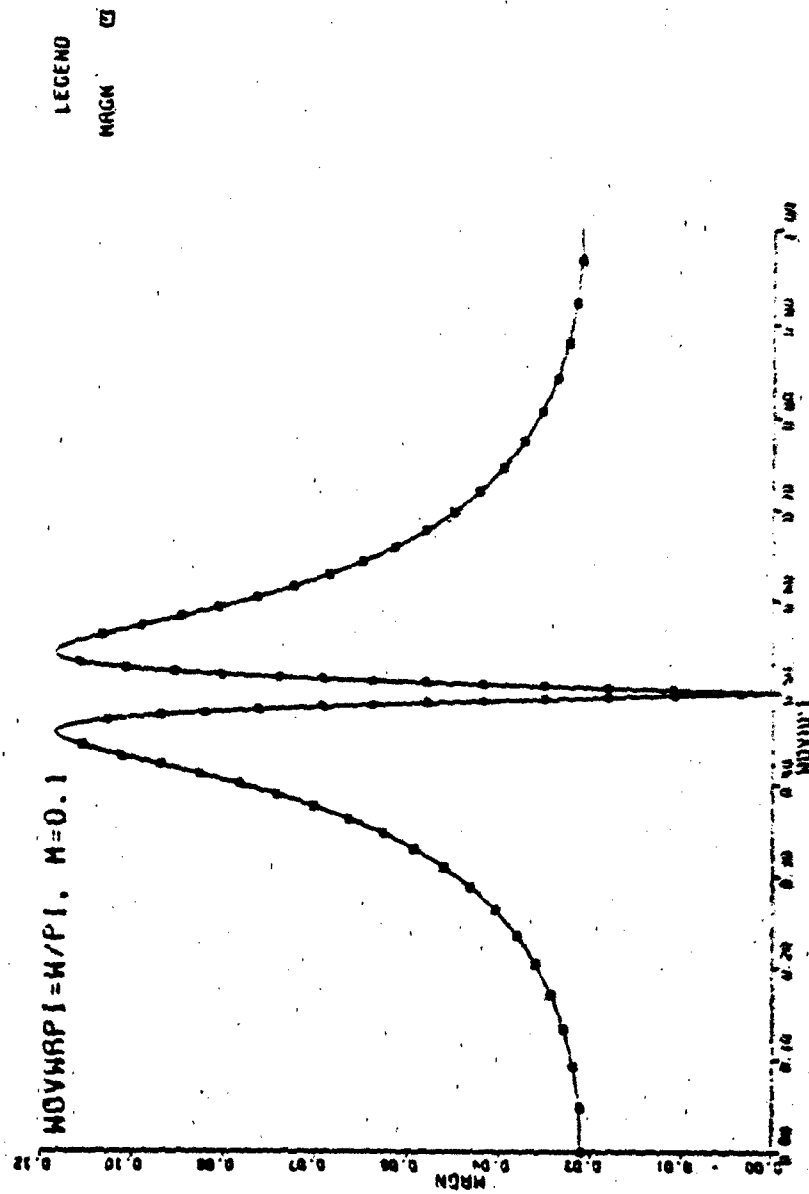


Figure 3.16. Filtered Signal-Jammer ( $\mu = 0.1$ )



#### IV. DUVALL BASED ADAPTIVE BEAMFORMER

##### A. BASIC CONFIGURATION

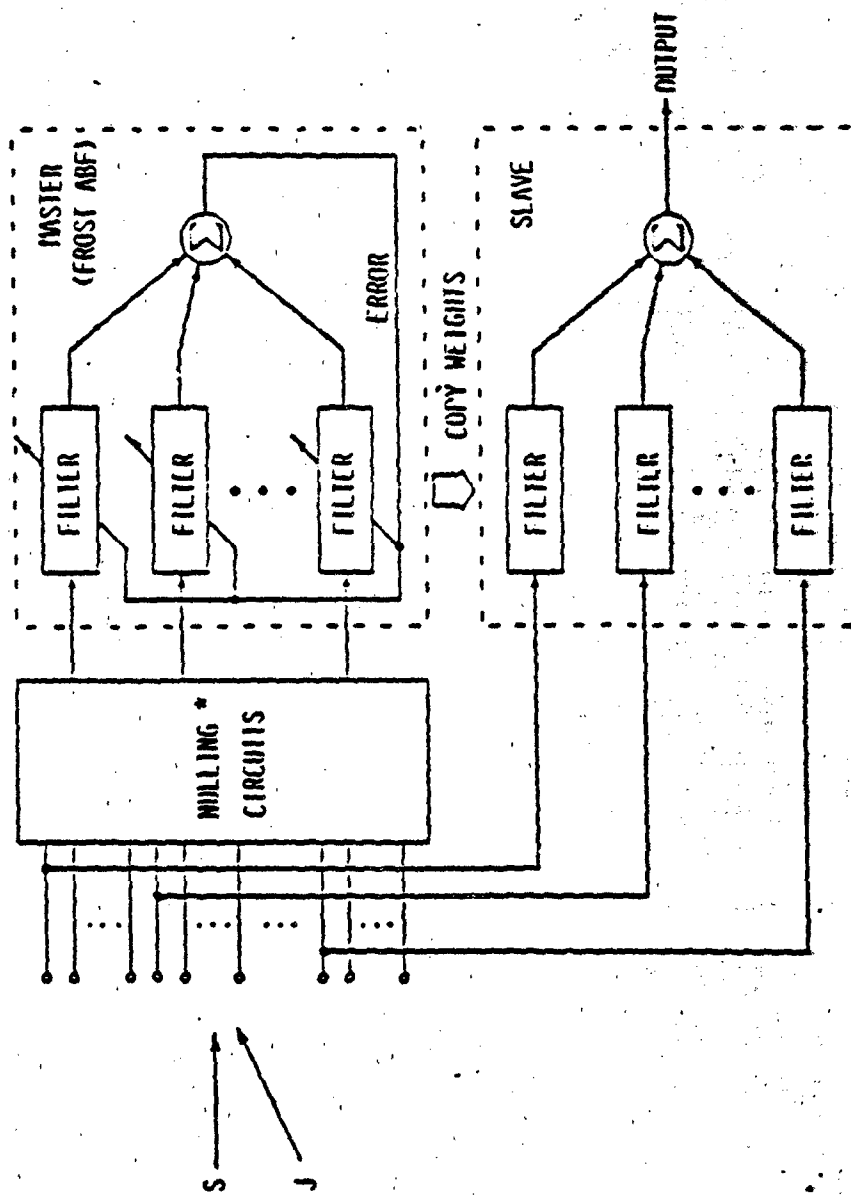
We have already illustrated some relatively simple signal/jammer scenarios and have seen that not only the Adaptive Noise Canceller (A.N.C.), but even the most rigid, constrained beamformers (Frost A.B.F.) can fail to preserve the desired signal from distortion.

In this chapter we will review and describe another solution to the noise cancellation problem and see how we can preserve the desired signal at the price of some increase in beamformer complexity.

The basic configuration of the DUVALL based beamformer [Ref. 5] is illustrated in Figure 4.1.

Two observations were made by Duvall useful in developing the Composite Beamformer (C.B.F.). The first observation was that interaction between the desired signal and the jammer is the root for the signal cancellation phenomenon. Frost experimentally and analytically has shown that the presence of both signal and jammer energy is a prerequisite for signal cancellation, and the output signal containing correlated signal and noise waveforms is the phenomenon that makes the signal distortion occur.

The second observation was that the signal plays no role in the optimum (i.e., Wiener) solution calculation in a



\*Nulls the desired signal

Figure 4.1. Basic Configuration of the DUVAL C.B.F.

perfectly steered Frost A.B.F.. In other words, the look-direction response is determined exclusively by the hard-constraints and not by the characteristics of the desired signal.

These two above observations were addressed for exclusion of the signal from the beamformer.

Reconsidering the whole structure of the C.B.F., we see that the key elements are:

1. An Augmented Array. The array has been augmented so that subarrays consisting of multiple elements appear in place of the individual elements of the original array.
2. A Preprocessor. A preprocessor operates upon the received signals from the augmented array to generate an environment that is free of desired-signal content.
3. An Adaptive Beamformer. The adaptive beamformer is required that can be constrained to control the look-direction response, while nulling all jamming signals.
4. A Slaved Non-Adaptive Beamformer. The adaptive beamformer derives weights that are copied to the slave beamformer, which has the same structure as the adaptive beamformer but is connected directly to selected antenna elements and is used to implement the computed solution and recover the desired signal.

Three of the above key elements, the array, the preprocessor, and the adaptive beamformer, afford considerable flexibility. A variety of specific realizations are possible. The slaved beamformer design is inflexible in the sense that it mirrors the design of the adaptive beamformer.

#### B. PRINCIPLES OF THE DUVALL C.B.F.

The general C.B.F. structure of Figure 4.1 can be specialized to yield the C.B.F. shown in Figure 4.2. Here

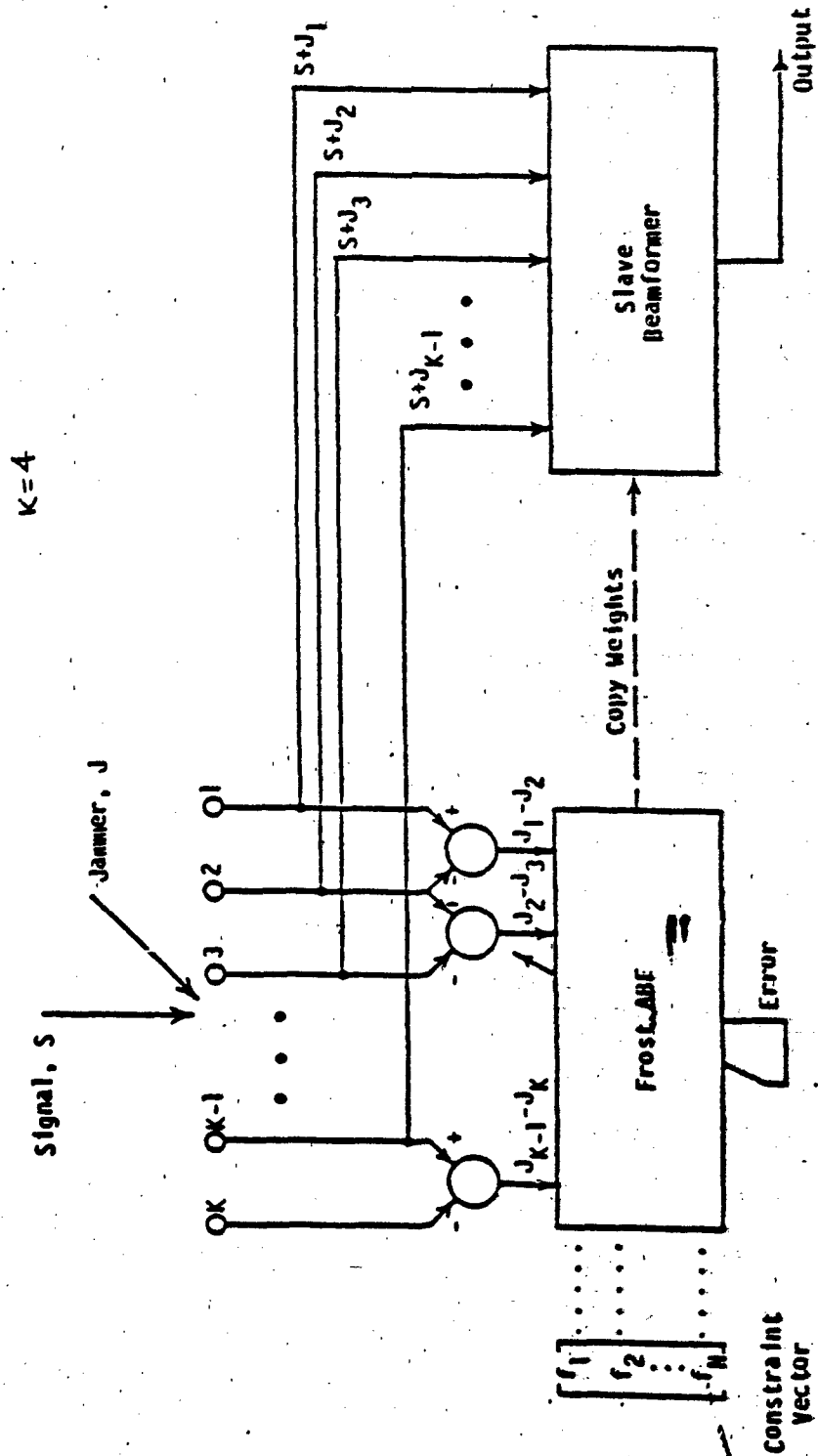


Figure 4.2. The DUVAL Composite Beamformer ( $K = 4$ )

the desired signal is assumed to be incident from broadside so we can neglect beam steering. The preprocessor is realized by using two-element subarrays in a simple element differencing scheme. The sharing of elements between subarrays provides very efficient use of elements; only a simple element is needed beyond those ordinarily required for a comparable array.

In order to prevent the desired signal from distortion (i.e., signal cancellation), the signal which arrives from the look-direction is excluded from the beamformer where the adaptive process takes place. Under the proposed scheme, the adaptive process is only used to determine a set of weights. These weights are copied into a separate, identical processor which is used to form the output signal. The C.B.F. scheme can be in cooperation with any existing adaptive algorithm including the Frost one, as Figure 4.2 addresses.

Due to the subtractive preprocessing, the look-direction signal does not appear at the Frost adaptive beamformer inputs, whose inputs are only composed of the jamming signal. The task of the Frost Algorithm is to place a null at the jammer direction through the determination of an appropriate set of weights and the signal has now no effect at all on the determination of those weights. By copying the weights the slaved processor places the main beam of the

antenna pattern to the constraints established by the look-direction and verifies that a null has already been placed in the jammer's direction.

Figure 4.3 represents the Duvall C.B.F. in terms of phasor notation. The jammer components received by the antenna are indicated by a set of equal amplitude uniformly-spaced phasors,  $J_0$ ,  $J_1$ ,  $J_2$ ,  $J_3$  and  $J_4$ . The phasor inputs to the Frost beamformer are  $J_1 - J_0$ ,  $J_2 - J_1$ ,  $J_3 - J_2$ , and  $J_4 - J_3$ . They are also uniform-amplitude, equally-spaced, and separated by the same angles as the received jammer components,  $J_0$ ,  $J_1$ ,  $J_2$ ,  $J_3$  and  $J_4$ .

Since the relative phase angles are the same in the slaved processor as in the Frost processor, the beam pattern notch is formed at the proper bearing angle.

The phasor argument applies to a single jammer at a single frequency. Linearity and superposition apply and show that phase relations are preserved for multiple jammers and for broadband as well as narrowband signals.

The uniform linear array provides an attractive structure for the C.B.F. because there is the option of element sharing between subarrays. A regular array structure is not, however, a prerequisite for the C.B.F.. The fundamental requirement is for phase matching between the master and the slave beamformers. Phase matching may be obtained for an arbitrary array geometry by augmenting each

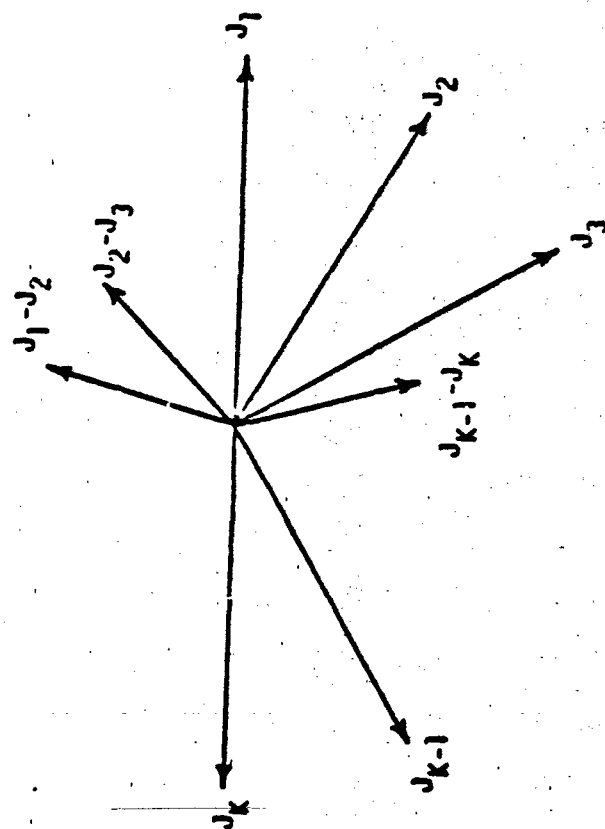


Figure 4.3. DUVALL C.B.F. Phasor Diagram

original element to form identical subarrays at the element locations. Identical preprocessors may then be used to form subarray responses with nulls in the selected look direction. The preprocessor outputs must be cophased for the look direction and applied to the adaptive beamformer while the original element outputs are cophased and applied to the slave.



## V. SPATIAL DITHER ALGORITHMS

### A. GENERAL IDEA

Spatial dither algorithms [Ref. 2] have been newly conceived for the purpose of applying locally controlled modulation to signals arriving at angles other than the look-direction, while leaving inputs arriving from the look-direction unchanged. The technique that these algorithms are employing is focused on a reduction of the jammer power density by spreading it spectrally. When the spatial dither is used with a conventional adaptive beamformer, it reduces the signal cancellation effects.

Conceptually, a simple form of spatial dither algorithm is represented in Figure 5.1 and stated as the "3/4-inch plywood" approach. The elements of an antenna array may be imagined to be fixed to a piece of plywood which provides rigid insulating support, so that the entire array may be moved mechanically. In either one or two dimensions, the array is moved in directions which are orthogonal to the look-direction. Far-field emanations arriving from the look-direction will be undistorted by the mechanical motion, while emissions from off axis sources will be distorted by an unusual shift-of-time-base form of modulation.

The outputs of the antenna elements of Figure 5.1 could be applied to a time delay and sum (nonadaptive) beamformer,

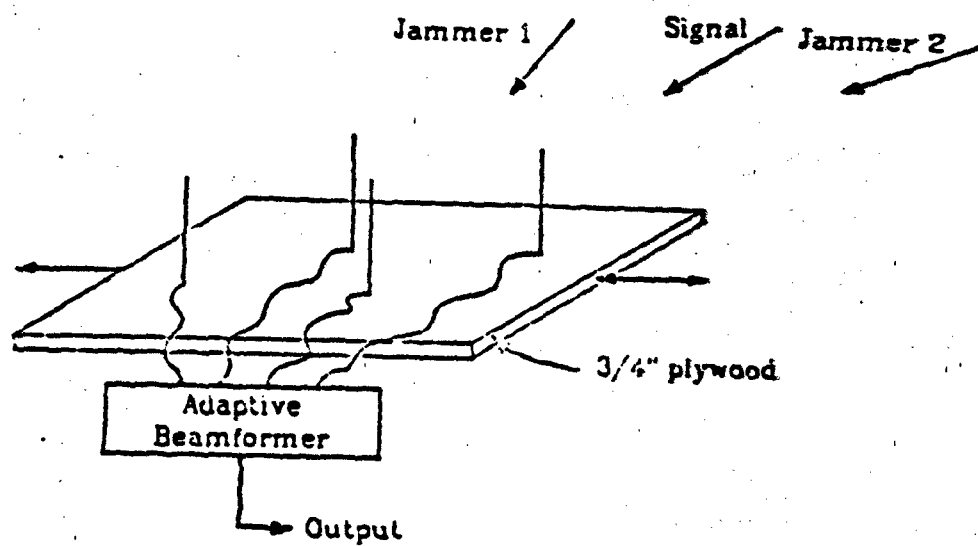


Figure 5.1. Spatial Dither Algorithm (3/4" Plywood Approach)

to a conventional adaptive beamformer, or to a Duvall adaptive beamformer. Spatial dither could be beneficial in each case. By reducing jammer power coherence, some anti-jam protection is provided without adaptive beamforming, and additional anti-jam protection is provided with adaptive beamforming. Reduction of signal cancellation effect in a Frost beamformer can be obtained by using spatial dither preprocessing. Breakup of jammer signal structure is a possible form of signal preprocessing applicable to all types of adaptive and nonadaptive beamformers.

## VI. CONCLUSIONS

Adaptive Noise Cancelling is a method of optimal filtering that can be applied whenever a suitable reference input is available. The principle advantages of the method that an A.N.C. employs are its adaptive capability, its low output noise, and its low signal distortion. The adaptive capability allows the processing of various signals whose properties are unknown. Output noise and signal distortion are generally lower than has been achieved with the various conventional optimal filtering configurations.

The FROST beamformer has proven itself good in rejecting various interference signals and enhancing the signal of interest for several cases. Interaction, though, between the desired signal and the various interference signals can lead to partial, or total, cancellation of the desired signal within the adaptive beamformer.

The DUVALL composite beamformer gives a solution to the problem of signal cancellation, at the price of increased complexity in the beamformer structure and implementation. Under that scheme, in order to prevent signal cancellation, the useful signal is excluded from the beamformer in which the adaptive process now takes place to derive a set of weights. These weights are now copied into a separate,

identical "slaved" processor used to form the output signal. The Ruvall approach to the problem is appealing but since it is a new development, possible limitations of its performance have not yet been assessed.

A WIDROW solution to the problem involves spatially, or electronically, moving the receiving array to modulate emanations received off the look-direction without distorting useful signals in the look-direction. This approach, called "spatial dither", introduces the additional possibility of modulating or smearing "smart" jammer signals, thereby limiting their effectiveness.

## APPENDIX A

### ANALYTICAL SOLUTION OF THE A.N.C.

Consider the two-weight noise canceller of Figure 2.3, and let us assume that the sampled reference inputs are:

$$x_{1j} = C \cos(\omega_0 jT + \phi) \quad (A.1)$$

$$x_{2j} = C \sin(\omega_0 jT + \phi) \quad (A.2)$$

Figure A.1 is a flow diagram showing signal propagation in such an adaptive noise canceller. The first step to the analysis is to obtain the isolated impulse response from the error,  $e$ , point C, to the filter output, point G, with the feedback loop from point G to point B broken.

Let an impulse of amplitude,  $a$ , to be applied at point C at discrete time,  $j = k$ ; that is,

$$e_j = a\delta(j - k) \quad (A.3)$$

The  $\delta(j - k)$  is a Kronecker delta function, defined as

$$\delta(j - k) = \begin{cases} 1 & \text{for } j = k \\ 0 & \text{otherwise.} \end{cases} \quad (A.4)$$

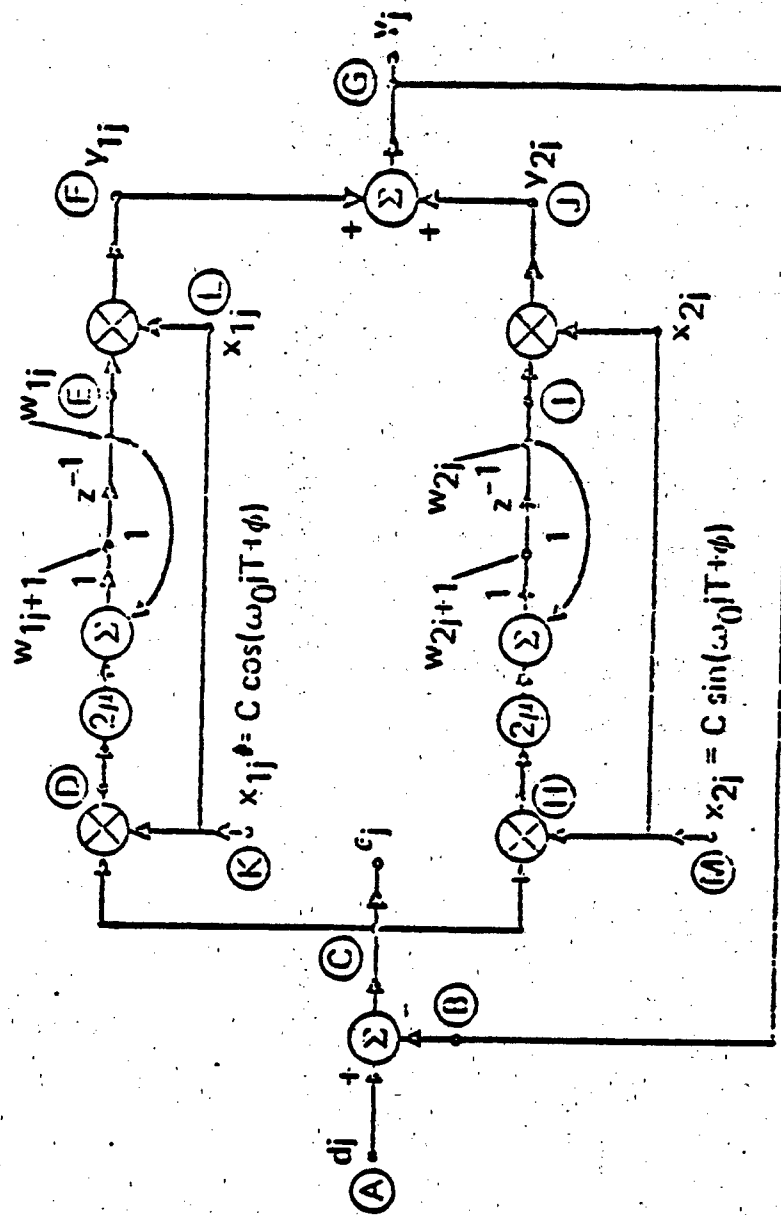


Figure A.1. Signal Propagation through an A.N.C.

The impulse causes a response at point D

$$\epsilon_j x_{1j} = \begin{cases} \alpha C \cos(\omega_0 kT + \phi) & \text{for } j = k \\ 0 & \text{otherwise.} \end{cases} \quad (\text{A.5})$$

which is the input scaled in amplitude by the instantaneous value of  $x_{1j}$  at  $j = k$ . The signal flow path from point D to point E is that of a digital integrator with transfer function,  $2u/(z-1)$ , and impulse response,  $2u(j-1)$ , where  $u(j)$  is the discrete unit step function

$$u(j) = \begin{cases} 0 & \text{for } j < 0 \\ 1 & \text{otherwise.} \end{cases} \quad (\text{A.6})$$

Convolving  $2u(j-1)$  with  $\epsilon_j x_{1j}$  yields a response at point E of

$$w_{1j} = 2 \alpha C \cos(\omega_0 kT + \phi) \quad (\text{A.7})$$

where  $j \geq k + 1$ . When the scaled and delayed step function is multiplied by  $x_{1j}$ , the response at point F is obtained as

$$y_{1j} = 2 \alpha C^2 \cos(\omega_0 jT + \phi) \cos(\omega_0 kT + \phi) \quad (\text{A.8})$$



where  $j \geq k + 1$ . The corresponding response at point J, obtained in a similar manner, is

$$y_{2J} = 2 \mu \alpha C^2 \sin(\omega_0 jT + \phi) \sin(\omega_0 kT + \phi) \quad (A.9)$$

where  $j \geq k + 1$ . Combining the preceding two equations yields the response at the filter output, point G:

$$\begin{aligned} y_{2J} &= 2 \mu \alpha C^2 \cos(\omega_0 T(j-k)) \\ &= 2 \mu \alpha C^2 u(j-k-1) \cos(\omega_0 T(j-k)) \end{aligned} \quad (A.10)$$

Note that the above is a function only of  $(j-k)$  and is thus a time invariant impulse response proportional to the input impulse.

A linear transfer function for the noise canceller may now be derived in the following manner. If the time,  $k$ , is set equal to zero, the unit impulse response of the linear time-invariant signal-flow path from point C to point G is

$$y_J = 2 \mu C^2 u(j-1) \cos(\omega_0 jT) \quad (A.11)$$

and the transfer function of this path is

$$G(z) = 2 \mu C^2 \left[ \frac{z(z - \cos \omega_0 T)}{z^2 - 2z \cos \omega_0 T + 1} - 1 \right]$$

$$= \frac{2\mu C^2 (z \cos \omega_0 T - 1)}{z^2 - 2z \cos \omega_0 T + 1} \quad (\text{A.12})$$

This function can be expressed in terms of a radian sampling frequency,  $\Omega = 2\pi/T$ , as

$$G(z) = \frac{2\mu C^2 [z \cos(2\pi\omega_0 \Omega^{-1}) - 1]}{z^2 - 2z \cos(2\pi\omega_0 \Omega^{-1}) + 1} \quad (\text{A.13})$$

If the feedback loop from point G to point B is now closed, the transfer function,  $H(z)$ , from the primary input, point A, to the noise canceller output, point C, can be obtained from the feedback formula

$$H(z) = \frac{z^2 - 2z \cos(2\pi\omega_0 \Omega^{-1}) + 1}{z^2 - 2(1 - \mu C^2)z \cos(2\pi\omega_0 \Omega^{-1}) + 1 - 2\mu C^2} \quad (\text{A.14})$$

This equation shows that the noise canceller with a cosine reference input has the properties of a notch filter at the reference frequency along the signal flow path from primary input to output.

## APPENDIX B

### TAPPED DELAY LINE FILTER

The tapped delay line [Ref. 5] is the preprocessor most often used in adaptive filtering applications. Most mathematical analyses of adaptive filters are based on a tapped delay line model. A tapped delay line can be implemented either in analog or digital form. The schematic of an adjustable filter based on an analog tapped delay line is shown in Figure B.1. Such a delay line has an analog input and the output at each tap is a delayed version of this input. Thus the  $n^{\text{th}}$  component of  $X(t)$  is the output at the  $n^{\text{th}}$  tap and is given by

$$x_n(t) = x(t - (n - 1)\Delta) \quad (\text{B.1})$$

where  $\Delta$  is the delay between taps and where

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \\ \vdots \\ x_N(t) \end{bmatrix} \quad (\text{B.2})$$

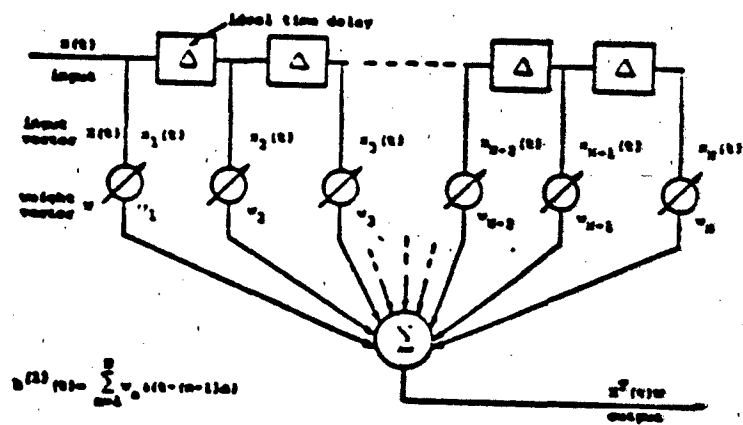


Figure B.1. Tapped Delay Line Filter

In digital implementation, the tapped delay line is implemented as a shift register either in hardware, Figure B.2, or by simulation inside the computer, Figure B.3. A new digital sample of the filter input is taken at every adaption cycle and shifted into the leftmost position of the shift register. Old samples are shifted right by one position and the oldest sample is shifted out altogether. Each position of the shift register is multiplied by the corresponding weight. Thus the digital output of the filter is the weighted sum of the last  $N$  samples of the input.

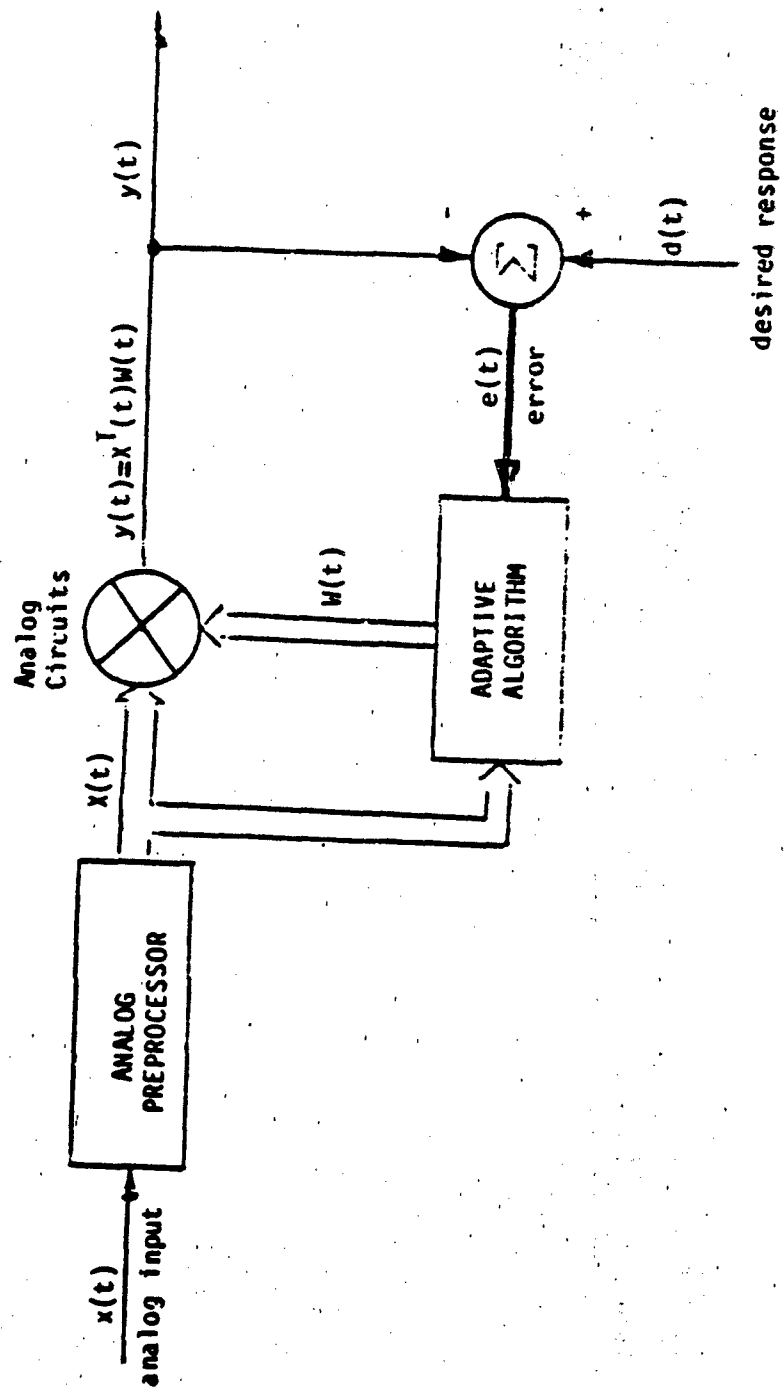


Figure 8.2. Digital Filter Type Adaptive Filter

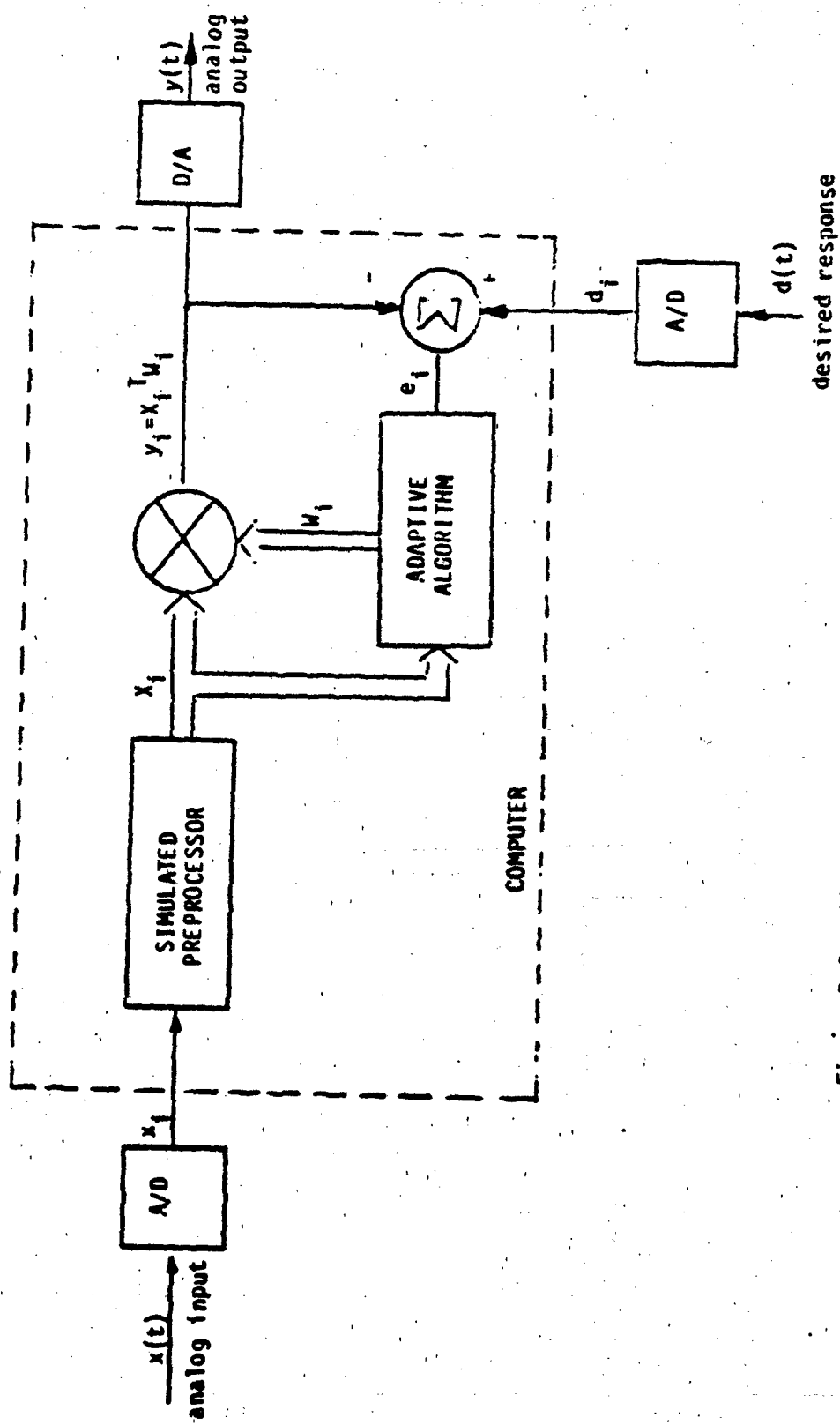


Figure B.3. Simulated Adaptive Filter

# APPENDIX C COMPUTER SOFTWARE UTILIZED

All plots through this work were obtained using Fortran and CSMP language programs running on the IBM 3033 main frame computer.

The following CSMP language program was used (for various different data) to produce the plots 2.5, 6, 7, 8, and 9. As inputs to the program are entered the coefficients of the transfer function  $H(z) = Y(z)/X(z)$  in the complex z-domain and as output is the plot of the magnitude versus frequency of the transfer function  $H(z)$ .

```
//MANIKAI JCB (2507,1286),'MANIKAS',CLASS=E
//*FCENAT PE,DDNAME=FICT.SYSVECTE,DEST=LOCAL
// EXEC CSMPXV
//X.SISLIB DD
//          DD
//          DD
//          DD
//          DD
CSH=SYS3.INSL.SP,DISF=SHR,VCL=SZE=MS003,UNIT=3350
//X.CCMPIINT DD CUMMY
//X.SISFIINT DD CUMMY
//X.FICTFASH DD *
      EPIC1 SCALE=.5 SEND
//X.SYSIN DE *
*****
*THIS PROGRAM COMPUTES THE FREQUENCY RESPONSE OF UP TO A*
*  TENTH ORDER FILTER WHOSE TRANSFER FUNCTION          *
*                CAN BE EXPRESSED AS:                    *
*
```



```

*      H(Z)=Y(Z)/X(Z)=N(Z)/D(Z),          WHERE      *
*
*      -1      -2      -3      -4      -10      *
*      N(Z)=A0+A1*Z +A2*Z +A3*Z +A4*Z + ... +A10*Z ,AND *
*
*      -1      -2      -3      -4      -10      *
*      D(Z)=E0+E1*Z +E2*Z +E3*Z +E4*Z + ... +E10*Z .    *
*
*      MAGNITUDE AND PHASE ARE GIVEN FOR THE INTERVAL      *
*      0 < W < PI ONLY SINCE THAT INFORMATION IS          *
*      REPEATED ON THE INTERVAL PI < W < 2*PI.            *
*      PHASE IS CONSTRAINED TO PLUS OR MINUS 180 DEGREES.   *
*      THE CORDINATE IS IN UNITS OF W/PI.                   *
*
*****
INITIAL
RENAME TIME=WOVRFI
PARAM FI=3.14159
PARAM M=0.001
PARAM W1=0.25
C1=-2.*COS(2.*PI*W1)
C2=-2.*(1.-M)*COS(2.*FI*W1)
C3=1.-2.*M
A1=C1
E1=C2
E2=C3
*****
*      ENTER THE 'A' COEFFICIENTS      *
*
PARAM A0= 1.0,A2= 1.,A3= 0.,A4= 0.,A5= 0.*
PARAM A6= 0.,A7= 0.,A8= 0.,A9= 0.,A10= 0.*
*****
*      ENTER THE 'E' COEFFICIENTS.      *
*
PARAM E0=1.0 ,E3= 0.,E4= 0.,E5= 0.      *
PARAM E6= 0.,E7= 0.,E8= 0.,E9= 0.,E10= 0.*
*****

```

```

DYNAMIC
NOSCHI
W=W0VRFPI*PI
DEGREE=180.*W0VRFPI
MAG1=A0+A1*COS(W)+A2*COS(W*2.)+A3*COS(W*3.) ...
      +A4*COS(W*4.)+A5*COS(W*5.)+A6*COS(W*6.) ...
      +A7*COS(W*7.)+A8*COS(W*8.) ...
      +A9*COS(W*9.)+A10*COS(W*10.)
MAG2=A1*SIN(W)+A2*SIN(W*2.)+A3*SIN(W*3.) ...
      +A4*SIN(W*4.)+A5*SIN(W*5.)+A6*SIN(W*6.) ...
      +A7*SIN(W*7.)+A8*SIN(W*8.) ...
      +A9*SIN(W*9.)+A10*SIN(W*10.)
MAG3=E0+E1*COS(W)+E2*COS(W*2.)+E3*COS(W*3.) ...
      +E4*COS(W*4.)+E5*COS(W*5.)+E6*COS(W*6.) ...
      +E7*COS(W*7.)+E8*COS(W*8.) ...
      +E9*COS(W*9.)+E10*COS(W*10.)
MAG4=E1*SIN(W)+E2*SIN(W*2.)+E3*SIN(W*3.) ...
      +E4*SIN(W*4.)+E5*SIN(W*5.)+E6*SIN(W*6.) ...
      +E7*SIN(W*7.)+E8*SIN(W*8.) ...
      +E9*SIN(W*9.)+E10*SIN(W*10.)
MAG=SQRT(MAG1**2+MAG2**2)/SQRT(MAG3**2+MAG4**2)
IF(MAG1.EQ.0.0.AND.MAG2.EQ.0.0) ANG1=0.
IF(MAG1.NE.0.0.OR.MAG2.NE.0.0) ...
  ANG1=ATAN2(MAG2,MAG1)*180./PI
IF(MAG3.EQ.0.0.AND.MAG4.EQ.0.0) ANG2=0.
IF(MAG3.NE.0.0.OR.MAG4.NE.0.0) ...
  ANG2=ATAN2(MAG4,MAG3)*180./PI
PHASE=ANG1-ANG2
IF(PHASE.GT.180.) PHASE=PHASE-360.
IF(PHASE.LE.-180.) PHASE=PHASE+360.
TIMES FINISH=1.0
PRINT W,DEGREE,MAG,PHASE
OUTPUT WCVRFPI,MAG
LABEL DIGITAL FREQUENCY RESPONSE - MAGNITUDE
LABEL OF THE NOTCH FILTER (W0VRFPI=W/PI,N=0.001)

```

```

PAGE XYFLOT
CUTPUT WCVREI,PHASE
LABEL FREQUENCY RESPONSE - PHASE IN DEGREES
LABEL OF THE NOTCH FILTER (WOVEPI=W/PI,M=0.001)
PAGE XYFLOT
END
STOP
ENDJCE

```

```

/*
/*
/*
/*

```

The following FCSTRAN program was used to produce the beam pattern of figure 3.5 .

```

C
C THIS PROGRAM DOES PCLAR PLOTTING OF A GIVEN POLAR
C FUNCTION (VTHE(I)) .VS. THE ANGLE (W(I))

```

```

      INTEGER I,J
      REAL W(73),MAG1(73),MAG2(73),
1      MAG3(73),MAG4(73),VTHE(73),
2      A0,A1,A2,EC,B1,B2,C1,C2,C3,
3      MAXV,PI,M,SI

```

```

C

```

```

      PI=3.14159
      M=.01
      W1=.125
      C1=-2.*COS(2.*PI*W1)
      C2=-2.*(1.-M)*COS(2.*PI*W1)
      C3=1.-2.*M
      A0=1.
      A1=C1
      A2=1.
      E0=0.
      B1=C2

```

```

      B2=C3
      DO 6 I = 1, 73
        W(I) = (FICAT(I) -1.0) *5.0*0.017453
6      CCNTINUE
      DO 7 I = 1, 73
        MAG1(I) =A0+A1*COS(W(I))+A2*COS(W(I)*2.)
        MAG2(I) =A1*SIN(W(I))+A2*SIN(W(I)*2.)
        MAG3(I) =B0+B1*COS(W(I))+B2*COS(W(I)*2.)
        MAG4(I) =B1*SIN(W(I))+B2*SIN(W(I)*2.)
7      CCNTINUE
      DO 10 I = 1, 73
        VTHE(I) =SQRT(MAG1(I)**2+MAG2(I)**2)/
$          SQRT(MAG3(I)**2+MAG4(I)**2)
        W(I) = W(I) * 57.296
10     CCNTINUE
C *** NORMALIZE AND FORM THE OUTPUT ARRAY VTHE(I) ***
C
      MAXV=VTHE(1)
      DO 12 I = 2, 73
        IF ((VTHE(I)).LE.(MAXV)) GO TO 12
        MAXV=VTHE(I)
12     CCNTINUE
      DO 13 I = 1, 73
        VTHE(I) = VTHE(I)/MAXV
13     CCNTINUE
C SET UP TEK618
      CALL TEK618
C      CALL COMRES
      CALL PAGE(8.5,11.)
      CALL BLOWUP(.6)
      CALL NOBEDR
      CALL AREA2D(7.5,7.5)
      CALL HEADIN ('BROADSIDE ARRAY PATTERN',-23,2.,1)
C SET UP POLAR GRID
      CALL POLAR(0.01745,.266,3.75,3.75)

```

CALL MARKER (16)  
CALL DOT  
CALL CURVE (W,VTHE,73,1)  
CALL RESET ('DOT')  
CALL GRID(2,4)  
CALL ENDPL(0)  
STOP  
END

#### LIST OF REFERENCES

1. J. R. Glover, Jr., Adaptive Noise Cancelling of Sinusoidal Interferences, Stanford University, Ph.D., 1975.
2. B. Widrow, et. al., Jamming of Adaptive Antennas by Signal Cancellation: The Phenomenon and Two Remedies, Department of Electrical Engineering, Stanford University, Stanford, California, n.d.
3. B. Widrow, et. al., "Adaptive Noise Cancelling: Principles and Applications," IEEE Proceedings, Vol. 63, No. 12, December 1975.
4. O. L. Frost, III, "An Algorithm for Linearly Constrained Adaptive Array Processing," IEEE Proceedings, Vol. 60, No. 8, pp. 926-935, August 1972.
5. K. M. Duvall, Signal Cancellation in Adaptive Antennas: The Phenomenon and a Remedy, Stanford University, Ph.D., 1983.
6. J. Kaunitz, Adaptive Filtering of Broadband Signals as Applied to Noise Cancelling, Stanford University, Ph.D., 1972.

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